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# Exact cross sections for the neutralino-slepton coannihilation

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Abstract: Coannihilation processes provide an important additional mechanism for reducing the density of stable relics in the Universe. In the case of the stable lightest neutralino of the MSSM, and in particular the Constrained MSSM (CMSSM), the coannihilation with sleptons plays a major role in opening up otherwise cosmologically excluded ranges of supersymmetric parameters. In this paper, we derive a full set of exact, analytic expressions for the coannihilation of the lightest neutralino with the sleptons into all two-body tree-level final states in the framework of minimal supersymmetry. We make no simplifying assumptions about the neutralino nor about sfermion masses and mixings other than the absence of explicit CP-violating terms and inter-family mixings. The expressions should be particularly useful in computing the neutralino WIMP relic abundance without the approximation of partial wave expansion. We illustrate the effect of our analytic results with numerical examples and demonstrate a sizeable difference with approximate expressions available in the literature.

Keywords: Supersymmetric Effective Theories, Cosmology of Theories beyond the SM, Dark Matter.




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## Contents

## 1. Introduction

The relic density of stable weakly-interacting massive particles (WIMPs) is determined primarily by how efficiently their number density in the early Universe can be reduced. In the case of the most popular WIMP candidate: the lightest neutralino of minimal supersymmetry (SUSY), assumed to be the lightest SUSY particle (LSP), there are two generic mechanisms [1, 2]. First, the neutralino can pair-annihilate into ordinary particles. Second, in some cases they can coannihilate with some some other species if these are nearly mass-degenerate with the LSPs.

The standard mechanism of neutralino pair-annihilation has been considered in much detail in many papers [2]. In particular, complete sets of neutralino annihilation cross sections were provided in [3] (see also [2]) in the case of partial wave approximation. Exact expressions applicable both near resonances and new final-state thresholds were recently published in [4].

The mechanism of coannihilation was originally pointed out by Griest and Seckel [5]. It applies when there exists some other species $\chi'$ which is not much heavier than the WIMP and may therefore be still present in the thermal plasma at the time of WIMP decoupling. Coannihilation becomes important if its annihilation with the WIMP (and/or itself) is equally, or more, efficient than the pair-annihilation of the stable WIMPs.

These circumstances in particular are naturally realized in the case of the higgsino-like lightest neutralino LSP in the framework of the Minimal Supersymmetric Standard Model (MSSM). In this case the next-to-lightest neutralino and the lightest chargino are almost mass-degenerate with the LSP [6, 7]. In fact, the coannihilation in this case is so efficient that it has a devastating effect on the relic density of the higgsino-like neutralino, which is the type of LSP strongly disfavored by naturalness [8] and mass-unification [9, 10, 11]. It even plays some role [7] in the strongly prefered case of the bino-like LSP [8].

Since, in the framework of general softly-broken low-energy SUSY, scalar superpartner masses are a priori unrelated (or at best loosely related) to the neutralino sector, in principle one might assume that any of the scalar superpartners could be light enough to participate in coannihilation with the neutralino LSP — the case that would be technically rather challenging and in any case not particularly well-motivated.

A more realistic scenario is the one in which one of the scalar partners of the top-quark or the $\tau$-lepton is rather light and nearly degenerate in mass with the neutralino LSP. This is because the off-diagonal elements in their $2 \times 2$ mass matrices can under some circumstances greatly reduce one of the eigenmasses relative to the other. The case of neutralino-stop coannihilation was considered in [12] in the framework of the MSSM and recently shown in [13] to be also applicable in the case of the Constrained MSSM (CMSSM) but only for rather large values of the trilinear soft SUSY-breaking term $A_0$.

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The importance of the neutralino coannihilation with the lighter of the two staus in the framework of the CMSSM was pointed out in [14]. In this model, when the common gaugino mass parameter $m_{1/2}$ is much larger than the common scalar mass parameter $m_0$, it is the lightest stau that is the LSP [11]. The effect of the neutralino-stau coannihilation is to open up a narrow corridor [14] just above the boundary of equal neutralino-stau masses into an otherwise cosmologically forbidden region in the $(m_{1/2}, m_0)$-plane. (Without coannihilation, the requirement of the relic abundance of the neutralino to be consistent with that allowed by the age of the Universe ($\Omega_\chi h^2 \lesssim \mathcal{O}(1)$) often provides a stringent upper bound on the parameters $m_{1/2}$ and $m_0$ [9, 11].) The effect has since been included in a number of recent analyses, e.g. in [15, 16], and in a publically available package for computing the relic density micrOMEGAs [17].

One should mention that there are two other ways of evading this otherwise generic cosmological bound on $m_{1/2}$ and $m_0$. One is realized for $m_0 \gg m_{1/2}$. In this region one invariably finds it difficult to satisfy the conditions of radiative electroweak symmetry breaking (EWSB); in other words the square of the Higgs/higgsino mass parameter $\mu$ comes out to be negative. In a very narrow corridor along the region of no-EWSB, $\mu$ grows rapidly from zero but it is still less than $m_{1/2}$ [11, 18]. As a result, the LSP has a sizeable higgsino component (although it is still mostly a bino, like in the rest of the $(m_{1/2}, m_0)$-plane) and its relic density is typically small. In fact, because of the growing LSP mass and its gaugino fraction, the relic density increases rapidly along a very steep slope from very small values, characteristic for lighther neutralinos with a larger higgsino admixture, to larger (and often cosmologically excluded) values characteristic of heavier and bino-dominated neutralino. As a result, the cosmologically favored range $0.1 < \Omega_\chi h^2 < 0.2$ is only realized there for a rather narrow range of $m_{1/2}$ [18, 19].

The other important escape route from the cosmological bound on $m_{1/2}$ and $m_0$ occurs when $\tan\beta$, which is the usual ratio of the vacuum expectation values of the neutral Higgs scalars, is large, $\gtrsim 50$. This is because the physical masses of the heavy Higgs scalar $H$ and pseudoscalar $A$ decrease with increasing $\tan\beta$ [3]. When the neutralino mass becomes large enough, close to half of the heavy Higgs boson mass, the LSP relic abundance becomes efficiently reduced through a relatively wide resonance involving mostly the pseudoscalar exchange. The effect is amplified by the coupling of $A$ to down-type fermions which grows like $\tan\beta$. (The heavy scalar Higgs coupling also grows in a similar fashion but its dominant contribution is only $p$-wave and therefore suppressed by square of the WIMP relative velocity.) In the framework of the CMSSM, the effect is to open up wide fractions of the otherwise cosmologically excluded ranges of the $(m_{1/2}, m_0)$-plane along the wide $A$-resonance [14], [19]–[22]. The precise position of the resonance shows a sizeable dependence on some input parameters, most notably on the ratio $m_t/m_b$, $A_0$, etc. and remains a subject of some ongoing debate. The full two-loop Higgs effective potential would have to be computed and implemented in the analysis to reduce the sensitivity to, e.g. the scale dependence.

In the CMSSM a set of reasonably well-motivated unification assumptions leads to only four parameters: a common gaugino mass $m_{1/2}$, a common scalar mass $m_0$, a trilinear coupling $A_0$, as well as $\tan\beta$. One is also free to choose the sign of the $\mu$-parameter, while its magnitude is determined by the mechanism of electroweak radiative symmetry breaking (EWSB). The CMSSM can therefore be considered as a well-motivated SUSY model with the smallest number of independent parameters. (In particular, the so-called minimal supergravity (mSUGRA) model can be viewed as a specific realization of the framework.) The CMSSM has become a benchmark model for the LHC and other SUSY searches.

As mentioned above, the neutralino-stau coannihilation effect is particularly important in the framework of the CMSSM, but it can affect the neutralino relic density also in the more general MSSM. This will be the framework in which we will work here for the sake of generality.

In this paper, we will present a full set of exact, analytic expressions for the tree-level cross section of the neutralino coannihilation with sleptons into all two-body final states in the general MSSM. In our analysis we will make no simplifying assumptions about the neutralino, nor will we assume the degeneracy of the left- and right-sfermion masses. We will not consider here the possibility of CP and flavor violation in the slepton sector although we will assume a general form of the left-right slepton mixing within each generation. We will include all tree-level final states and all intermediate states. We will also keep finite widths in $s$-channel resonances. A set of expressions for the neutralino-slepton coannihilaton was given in [14, 23] but only in the approximation of the partial wave expansion. Furthermore, these formulae did not include the effects of the tau Yukawa, of the $\widetilde{\tau}_1 - \widetilde{\tau}_2$ mixing and in some channels of the mass of the $\tau$, etc. which make them less reliable at large $\tan\beta \gtrsim 20$ [24].

The results presented here are exact, include all the terms and are valid both small and large values of $\tan\beta$, and both near and away from resonances and thresholds for new final states. This paper is meant to be a follow-up to [4] where we have calculated all the analytic cross sections of all tree-level processes for the neutralino pair-annihilation into all two-body final states. We follow the same conventions and notations as in [4].

The plan of the paper is as follows. In section 2 we briefly review the formalism for computing the relic density in the presence of both annihilation and coannihilation. In section 3 we introduce the relevant ingredients of the MSSM and list all the neutralino pair-annihilation channels. Explicit expressions for the coannihilation cross secion are given in section 4. In section 5 we present some numerical examples and in section 6 we summarize our work. Appendix A contains a complete list of relevant couplings and in appendix B we provide expressions for several auxiliary functions used in the text.

## 2. Calculation of the relic density with coannihilation

As summarized, e.g. in [4], the time evolution and subsequent freeze-out of a stable relic in an expanding Universe are described by the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma v_{\mathrm{M\o l}} \rangle \left[ n^2 - (n^{\mathrm{eq}})^2 \right], \tag{2.1}$$

where $n$ stands for the species' number density, $n^{\mathrm{eq}}$ is the number density that it would have had if it had remained in thermal equilibrium, $H(T)$ is the Hubble expansion rate,

$\sigma$ denotes the cross section of the species pair-annihilation into all allowed final states, $v_{\mathrm{Møl}}$ is the so-called Møller velocity [25] which is effectively the relative velocity of the two initial-state non-relativistic particles in the CM frame, and $\langle \sigma v_{\mathrm{Møl}} \rangle$ represents the thermal average of $\sigma v_{\mathrm{Møl}}$. In the early Universe, the species were initially in thermal equilibrium, $n = n^{\mathrm{eq}}$. The number density decreased with the expanding Universe and the relic froze out of the thermal equilibrium when its typical interaction rate became less than the Hubble parameter.

In a more general framework with coannihilation, the stable particle $\chi$ is the lightest of $N$ species $\chi_i$, each with mass $m_i$, number density $n_i$, equilibrium number density $n_i^{\mathrm{eq}}$ and the number of internal degrees of freedom $g_i$. Griest and Seckel [5] showed that the total number density of all the species taking part in the coannihilation process

$$n = \sum_i^N n_i \tag{2.2}$$

obeys the Boltzmann equation as given in eq. (2.1). The quantity $\langle \sigma v_{\mathrm{Møl}} \rangle$ now stands for

$$\langle \sigma v_{\mathrm{Møl}} \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\mathrm{eq}}}{n^{\mathrm{eq}}} \frac{n_j^{\mathrm{eq}}}{n^{\mathrm{eq}}} , \tag{2.3}$$

where

$$\sigma_{ij} = \sigma \left( \chi_i \chi_j \longrightarrow \mathrm{all} \right) \tag{2.4}$$

and $v_{ij}$ are the relative (Møller) velocities of the coannihilating particles. Edsjö and Gondolo [7] recast eq. (2.3) into a convenient form involving only one-dimensional integrals

$$\langle \sigma v_{\mathrm{Møl}} \rangle = \frac{\int_{4m_\chi^2}^{\infty} ds\, s^{3/2} K_1 \left( \frac{\sqrt{s}}{T} \right) \sum_{ij}^N \beta_f^2(s, m_i, m_j) \frac{g_i g_j}{g_\chi^2} \sigma_{ij}(s)}{8 m_\chi^4 T \left[ \sum_i^N \frac{g_i}{g_\chi} \frac{m_i^2}{m_\chi^2} K_2^2(m_i/T) \right]^2} , \tag{2.5}$$

where $s = (p_i + p_j)^2$ is the usual Mandelstam variable, $K_i$ denotes the modified Bessel function of order $i$, and the kinematic factor $\beta_f(s, m_i, m_j)$ is given by

$$\beta_f(s, m_i, m_j) = \left[ 1 - \frac{(m_i + m_j)^2}{s} \right]^{1/2} \left[ 1 - \frac{(m_i - m_j)^2}{s} \right]^{1/2} . \tag{2.6}$$

Note that eq. (2.6) reduces to a familiar expression derived by Gondolo and Gelmini [25] in the case of single particle annihilation. (Compare, e.g. [4, eq. (2.3)].) Note also that, because of the assumed $R$-parity, after the freeze-out only the LSP $\chi$ will survive and its number density will at the end be $n_\chi = n$.

Equations (2.1) and (2.5) can be rather accurately solved by iteratively solving the equation

$$x_f^{-1} = \ln \left[ \frac{m_\chi}{2\pi^3} \frac{g_{\mathrm{eff}}}{2} \sqrt{\frac{45}{2 g_* G_N}} \langle \sigma v_{\mathrm{Møl}} \rangle (x_f)\, x_f^{1/2} \right] , \tag{2.7}$$

where $g_{\mathrm{eff}}$ is the effective number of degrees of freedom of the coannihilating particles [5],

$g_*$ represents the effective number of degrees of freedom at freeze-out ($\sqrt{g_*} \simeq 9$), $G_N$ is the gravitational constant, $x = T/m_\chi$ and the freeze-out point $x_f \equiv T_f/m_\chi$ is roughly 1/25 to 1/20. (See, e.g. [26].)

The neutralino LSP relic abundance is $\Omega_\chi h^2 = \rho_\chi/\rho_{\rm crit}$, where the critical density is $\rho_{\rm crit} = 1.05 \times 10^{-5}\,(h^2)$ GeV$/cm^3$, and $\rho_\chi$ today is to a good approximation given by

$$\rho_\chi = \frac{1.66}{M_{\rm Pl}} \left(\frac{T_\chi}{T_\gamma}\right)^3 T_\gamma^3 \sqrt{g_*} \left[\int_0^{x_f} dx \langle \sigma v_{\rm Møl}\rangle(x)\right]^{-1}, \tag{2.8}$$

where $M_{\rm Pl} = 1/\sqrt{G_N}$ denotes the Planck mass, $T_\chi$ and $T_\gamma$ are the present temperatures of the neutralino and the photon, respectively. The suppression factor $(T_\chi/T_\gamma)^3 \approx 1/20$ follows from entropy conservation in a comoving volume [27].

## 3. WIMP coannihilation in the MSSM

We will be working in the framework of the general MSSM. (For a review, see, e.g. [28]. We follow the conventions of [29].) We will not define it here but instead refer the reader to our previous publication [4] where also all the relevant quantities are introduced. All the remaining couplings that we will need here are summarized in appendix A.

In this section we summarize all the coannihilation channels of the neutralino LSP with the sleptons into tree-level two-body final states. They are listed in table 1. We only neglect slepton coannihilation with the lightest chargino and next-to-lightest neutralino which might be of some importance in the higgsino-dominant case. However, in this region the neutralino-chargino coannihilation is very effective anyway in reducing the relic density below any interesting level.

We further neglect (s-)lepton flavor (generation) mixing, but include the left-right mixing for the sleptons. Thus $a = 1, 2$, where the index $a$ denotes the slepton mass state within each family. $\ell$ and $\ell'$ represent charged leptons of different generations.

## 4. Exact expressions

We now move on to present a full set of exact, analytic expressions for the total cross sections for the neutralino-slepton coannihilation processes. We have included all contributing diagrams as well as all interference terms and kept finite widths of all $s$-channel resonances. In addition to neglecting CP violating phases in SUSY parameters we have also assumed no mixing among different generations of leptons and sleptons, although we have kept the left-right slepton mixing within each generation. The results presented here are meant to supplement and extend the ones presented in our previous paper [4] for the case of the neutralino-pair annihilation.

In the presence of coannihilation the formalism used in [4] must be generalized to include different initial states. First, following [7, 23, 30] we introduce a Lorentz-invariant function $w_{ij}(s)$

$$w_{ij}(s) = \frac{1}{4} \int d\,\mathrm{LIPS}\, |\mathcal{A}(ij \longrightarrow \mathrm{all})|^2 , \tag{4.1}$$

| Process | Exchanged particles | | | |
|---|---|---|---|---|
| | $s$-channel | $t$-channel | $u$-channel | PI |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \longrightarrow WW$ | $h, H, \gamma, Z$ | | $\widetilde{\nu}_\ell$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to ZZ$ | $h, H$ | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to Z\gamma$ | | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma\gamma$ | | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to W^\pm H^\mp$ | $h, H, A$ | | $\widetilde{\nu}_\ell$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to Zh, ZH$ | $A, Z$ | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to ZA$ | $h, H$ | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma h, \gamma H$ | | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma A$ | | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to hh, HH, hH$ | $h, H$ | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to hA, HA$ | $A, Z$ | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to AA$ | $h, H$ | $\widetilde{\ell}_c$ | $\widetilde{\ell}_c$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to H^+H^-$ | $h, H, \gamma, Z$ | | $\widetilde{\nu}_\ell$ | 4P |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \ell\bar{\ell}$ | $h, H, A, \gamma, Z$ | $\chi_i^0$ | | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to q\bar{q}$ | $h, H, A, \gamma, Z$ | | | |
| $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \nu_\ell\bar{\nu}_\ell$ | $Z$ | $\chi_k^\pm$ | | |
| $\widetilde{\ell}_a\widetilde{\ell}_b'^* \to \ell\bar{\ell}'$ | | $\chi_i^0$ | | |
| $\widetilde{\ell}_a\widetilde{\ell}_b'^* \to \nu_\ell\bar{\nu}_{\ell'}$ | | $\chi_k^\pm$ | | |
| $\widetilde{\ell}_a\widetilde{\ell}_b \to \ell\ell$ | | $\chi_i^0$ | $\chi_i^0$ | |
| $\widetilde{\ell}_a\widetilde{\ell}_b' \to \ell\ell'$ | | $\chi_i^0$ | | |
| $\widetilde{\ell}_a\chi \to Z\ell$ | $\ell$ | $\widetilde{\ell}_c$ | $\chi_i^0$ | |
| $\widetilde{\ell}_a\chi \to \gamma\ell$ | $\ell$ | $\widetilde{\ell}_c$ | | |
| $\widetilde{\ell}_a\chi \to W^-\nu_\ell$ | $\ell$ | $\widetilde{\nu}_\ell$ | $\chi_k^\pm$ | |
| $\widetilde{\ell}_a\chi \to h\ell, H\ell$ | $\ell$ | $\widetilde{\ell}_c$ | $\chi_i^0$ | |
| $\widetilde{\ell}_a\chi \to A\ell$ | $\ell$ | $\widetilde{\ell}_c$ | $\chi_i^0$ | |
| $\widetilde{\ell}_a\chi \to H^-\nu_\ell$ | $\ell$ | $\widetilde{\nu}_\ell$ | $\chi_k^\pm$ | |

**Table 1:** A complete set of processes relevant for the neutralino-slepton coannihilation into tree-level two-body final states in the MSSM. "PI" denotes four-point (4P) interactions. The notation is as follows: $\widetilde{\ell}_a$ denotes $\widetilde{e}_a$, $\widetilde{\mu}_a$ and $\widetilde{\tau}_a$, where $a = 1, 2$ (and likewise $b$ and $c$) denotes a slepton mass state index for each generation. The symbols $\ell$ and $\ell'$ represent charged leptons of different generations, and likewise for the sleptons. For the neutralino index $i = 1, \ldots, 4$ and for the chargino one $k = 1, 2$. Note that for each reaction with a non-zero net electric charge there is a corresponding one with the the opposite net charge which is not included in the Table. For example, in addition to $\widetilde{\ell}_a\widetilde{\ell}_b \to \ell\ell$ there is also $\widetilde{\ell}_a^*\widetilde{\ell}_b^* \to \bar{\ell}\bar{\ell}$.

where $|\mathcal{A}(ij \to \text{all})|^2$ denotes the absolute square of the reduced matrix element for the annihilation of two initial particles $ij$ into all allowed final states, averaged over initial spins and summed over final spins. The function $w_{ij}(s)$ is related to the coannihilation cross section $\sigma_{ij}(s)$ in eq. (2.5) via [7, 14]

$$w_{ij}(s) = \frac{s}{2}\beta_f(s, m_i, m_j)\, \sigma_{ij}(s)\,. \tag{4.2}$$

The expression (4.2) readily reduces to a more familiar form in the case of single-particle annihilation. (Compare, e.g. eq. (4.2) of [4].)

Since $w_{ij}(s)$ receives contributions from all the kinematically allowed annihilation process $ij \to f_1 f_2$, it can be written as

$$w_{ij}(s) = \frac{1}{32\,\pi} \sum_{f_1 f_2} \left[ c\theta \left( s - (m_{f_1} + m_{f_2})^2 \right) \beta_f\left(s, m_{f_1}, m_{f_2}\right)\, \widetilde{w}_{ij \to f_1 f_2}(s) \right], \tag{4.3}$$

where the summation extends over all possible two-body final states $f_1 f_2$, $m_{f_1}$ and $m_{f_2}$ denote their respective masses, and

$$c = \begin{cases} c_f\,, & \text{if } f_1(2) = f(\bar{f})\,, \\ 1\,, & \text{otherwise}\,, \end{cases} \tag{4.4}$$

where $c_f$ is the color factor of SM fermions ($c_f = 3$ for quarks and $c_f = 1$ for leptons).

Since $w_{ij}$ are Lorentz-invariant functions, we choose, for convenience, the CM frame in which the function $\widetilde{w}_{ij \to f_1 f_2}(s)$ can be expressed as

$$\widetilde{w}_{ij \to f_1 f_2}(s) = \frac{1}{2} \int_{-1}^{+1} d\cos\theta_{CM}\, |\mathcal{A}(ij \longrightarrow f_1 f_2)|^2\,, \tag{4.5}$$

where $\theta_{CM}$ denotes the scattering angle in the CM frame. In other words, we write $|\mathcal{A}(ij \to f_1 f_2)|^2$ as a function of $s$ and $\cos\theta_{CM}$, which greatly simplifies the computation.

We will follow table 1 in presenting explicit expressions for $\widetilde{w}_{ij \to f_1 f_2}(s)$ for all the two-body final states. All the couplings are defined in appendix A. All other auxiliary functions, are listed in appendix B. Some symbols are obvious (e.g. $m_W$ is the mass of the W-boson) and will not be defined here.

We begin by presenting the results for $\widetilde{\ell}_a \widetilde{\ell}_b^* \to$ vector boson pairs: $WW$, $ZZ$, $Z\gamma$ and $\gamma\gamma$.

### 4.1 $\widetilde{\ell}_a \widetilde{\ell}_b^* \to WW$

This process involves the $s$-channel CP-even Higgs boson ($h$ and $H$) exchange, the four-point interaction, the $s$-channel Z-boson and photon exchange, and the $u$-channel sneutrino ($\widetilde{\nu}_\ell$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a \widetilde{\ell}_b^* \to WW} = \widetilde{w}_{WW}^{(h,H,P)} + \widetilde{w}_{WW}^{(Z,\gamma)} + \widetilde{w}_{WW}^{(\widetilde{\nu})} + \widetilde{w}_{WW}^{(h,H,P-\widetilde{\nu})} + \widetilde{w}_{WW}^{(Z,\gamma-\widetilde{\nu})}\ : \tag{4.6}$$

- Higgs $(h, H)$ exchange (+ Point interaction):

$$\widetilde{w}_{WW}^{(h,H,P)} = \left| \sum_{r=h,H} \frac{C^{WWr} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\widetilde{\ell}_b \widetilde{\ell}_a WW} \right|^2 \frac{s^2 - 4m_W^2 s + 12 m_W^4}{4 m_W^4}\,; \tag{4.7}$$

- $Z,\gamma$ exchange:

$$\widetilde{w}_{WW}^{(Z,\gamma)} = \left| \frac{C^{WWZ} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} + \frac{e^2 \delta_{ab}}{s} \right|^2 \left\{ s - \left( m_{\widetilde{\ell}_a} + m_{\widetilde{\ell}_b} \right)^2 \right\} \times$$
$$\times \left\{ s - \left( m_{\widetilde{\ell}_a} - m_{\widetilde{\ell}_b} \right)^2 \right\} \cdot \frac{s^3 + 16 m_W^2 s^2 - 68 m_W^4 s - 48 m_W^6}{12 s m_W^4} ; \tag{4.8}$$

- sneutrino ($\widetilde{\nu}_\ell$) exchange:

$$\widetilde{w}_{WW}^{(\widetilde{\nu})} = \frac{1}{m_W^4} \left| C^{\widetilde{\ell}_b^* \widetilde{\nu}_\ell W^-} C^{\widetilde{\nu}_\ell^* \widetilde{\ell}_a W^+} \right|^2 \times$$
$$\times \Bigg[ \mathcal{T}_4 - 2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 2 m_W^2 \right) \mathcal{T}_3 +$$
$$+ \left\{ m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 + 4 m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 + 2 m_W^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 3 m_W^2 \right) \right\} \mathcal{T}_2 -$$
$$- 2 \Big\{ \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) \left( m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 - m_W^4 \right) +$$
$$+ m_W^2 \left( m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 - 4 m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 + 2 m_W^4 \right) \Big\} \mathcal{T}_1 +$$
$$+ \left( m_{\widetilde{\ell}_a}^2 - m_W^2 \right)^2 \left( m_{\widetilde{\ell}_b}^2 - m_W^2 \right)^2 \mathcal{T}_0 \Bigg] , \tag{4.9}$$

where the auxiliary functions $\mathcal{T}_i$ $(i = 0, 1, 2, 3, 4)$ are defined in appendix B. The arguments for these functions in the above should be understood as $\mathcal{T}_i = \mathcal{T}_i(s, m_{\widetilde{\ell}_a}^2, m_{\widetilde{\ell}_b}^2, m_W^2, m_W^2, m_{\widetilde{\nu}_\ell}^2, m_{\widetilde{\nu}_\ell}^2)$. Throughout the text, the second and the third arguments denote the masses for the two initial particles. The fourth and the fifth ones correspond to the masses for the two final particles. The last two arguments are the masses for the exchanged particles in the relevant $t$- and/or $u$-channel diagrams. Assuming this convention, we will omit the arguments for $\mathcal{T}_i$ in the following.

- Higgs $(h, H)$ (+ Point) - sneutrino ($\widetilde{\nu}_\ell$) interference:

$$\widetilde{w}_{WW}^{(h,H,P-\widetilde{\nu})} = \frac{1}{2 m_W^4} \mathrm{Re} \left[ \left( \sum_{r=h,H} \frac{C^{WWr} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\widetilde{\ell}_b^* \widetilde{\ell}_a WW} \right)^* C^{\widetilde{\ell}_b^* \widetilde{\nu}_\ell W^-} C^{\widetilde{\nu}_\ell^* \widetilde{\ell}_a W^+} \right] \times$$
$$\times \Bigg[ s^2 + s \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2 m_{\widetilde{\nu}_\ell}^2 - 4 m_W^2 \right) + 2 m_W^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2 m_{\widetilde{\nu}_\ell}^2 + 2 m_W^2 \right) -$$
$$- 2 \Big[ s \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\nu}_\ell}^2 - m_W^2 \right) \left( m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\nu}_\ell}^2 - m_W^2 \right) +$$
$$+ 2 m_W^2 \Big\{ m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 - m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 - (m_{\widetilde{\nu}_\ell}^2 + m_W^2) \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) +$$
$$+ (m_{\widetilde{\nu}_\ell}^2 - m_W^2)^2 \Big\} \Big] \mathcal{F} \Bigg] , \tag{4.10}$$

where the auxiliary function $\mathcal{F}$ is defined in appendix B. The arguments for these functions in the above should be understood as $\mathcal{F} = \mathcal{F}(s, m_{\widetilde{\ell}_a}^2, m_{\widetilde{\ell}_b}^2, m_W^2, m_W^2, m_{\widetilde{\nu}_\ell}^2)$. Throughout the text, the second and the third arguments of $\mathcal{F}$ denote the masses for the two initial particles. The fourth and the fifth ones correspond to the masses for the two final particles. The last argument is the mass for the exchanged particles in the relevant $t$- and/or $u$-channel diagrams. With this understanding, we will omit the arguments for $\mathcal{F}$ in the following.

- $Z, \gamma$ - sneutrino ($\widetilde{\nu}_\ell$) interference:

$$\begin{aligned}
\widetilde{w}_{WW}^{(Z,\gamma-\widetilde{\nu})} &= \frac{1}{6m_W^4}\,\mathrm{Re}\left[\left(\frac{C^{WWZ}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z}+\frac{e^2\delta_{ab}}{s}\right)^* C^{\widetilde{\ell}_b^*\widetilde{\nu}_\ell W^-}C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a W^+}\right]\times \\
&\quad \times\left[I_1^{WW}+6I_2^{WW}\mathcal{F}\right], \qquad (4.11)
\end{aligned}$$

where

$$\begin{aligned}
I_1^{WW} &= -s^3+2s^2\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2-9m_W^2\right)+ \\
&\quad +s\Big\{-m_{\widetilde{\ell}_a}^4-m_{\widetilde{\ell}_b}^4-10m_{\widetilde{\ell}_a}^2m_{\widetilde{\ell}_b}^2+12\left(m_{\widetilde{\nu}_\ell}^2+2m_W^2\right)\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)+ \\
&\qquad +4\left(-3m_{\widetilde{\nu}_\ell}^4+6m_W^2m_{\widetilde{\nu}_\ell}^2+7m_W^4\right)\Big\}+ \\
&\quad +2m_W^2\Big\{-9\left(m_{\widetilde{\ell}_a}^4+m_{\widetilde{\ell}_b}^4\right)+30m_{\widetilde{\ell}_a}^2m_{\widetilde{\ell}_b}^2- \\
&\qquad -4\left(3m_{\widetilde{\nu}_\ell}^2+m_W^2\right)\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)+12\left(m_{\widetilde{\nu}_\ell}^2-m_W^2\right)^2\Big\}- \\
&\quad -\frac{8}{s}m_W^4\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2, \qquad (4.12) \\
I_2^{WW} &= -s^2\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\nu}_\ell}^2-m_W^2\right)\left(m_{\widetilde{\ell}_b}^2-m_{\widetilde{\nu}_\ell}^2-m_W^2\right)+ \\
&\quad +s\Big\{m_{\widetilde{\ell}_a}^2m_{\widetilde{\ell}_b}^2\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)-\left(m_{\widetilde{\nu}_\ell}^2+3m_W^2\right)\left(m_{\widetilde{\ell}_a}^4+m_{\widetilde{\ell}_b}^4\right)- \\
&\qquad -2\left(2m_{\widetilde{\nu}_\ell}^2+m_W^2\right)m_{\widetilde{\ell}_a}^2m_{\widetilde{\ell}_b}^2+\left(3m_{\widetilde{\nu}_\ell}^4+5m_W^4\right)\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)- \\
&\qquad -2\left(m_{\widetilde{\nu}_\ell}^6-4m_W^2m_{\widetilde{\nu}_\ell}^4+m_W^4m_{\widetilde{\nu}_\ell}^2+2m_W^6\right)\Big\}+ \\
&\quad +2m_W^2\Big\{\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)\left(m_{\widetilde{\ell}_a}^4+m_{\widetilde{\ell}_b}^4-3m_{\widetilde{\ell}_a}^2m_{\widetilde{\ell}_b}^2\right)- \\
&\qquad -\left(m_{\widetilde{\nu}_\ell}^2+m_W^2\right)\left(m_{\widetilde{\ell}_a}^4+m_{\widetilde{\ell}_b}^4\right)+4m_{\widetilde{\ell}_a}^2m_{\widetilde{\ell}_b}^2\left(2m_{\widetilde{\nu}_\ell}^2+m_W^2\right)- \qquad (4.13) \\
&\qquad -\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)\left(m_{\widetilde{\nu}_\ell}^2-m_W^2\right)\left(3m_{\widetilde{\nu}_\ell}^2+m_W^2\right)+2\left(m_{\widetilde{\nu}_\ell}^2-m_W^2\right)^3\Big\}.
\end{aligned}$$

### 4.2 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to ZZ$

This process proceeds via the $s$-channel CP-even Higgs boson ($h$ and $H$) exchange, the four-point interaction, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to ZZ} = \widetilde{w}_{ZZ}^{(h,H,P)} + \widetilde{w}_{ZZ}^{(\widetilde{\ell})} + \widetilde{w}_{ZZ}^{(h,H,P-\widetilde{\ell})} \quad : \qquad (4.14)$$

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- Higgs $(h, H)$ exchange (+ Point interaction):

$$\widetilde{w}_{ZZ}^{(h,H,P)} = \left| \sum_{r=h,H} \frac{C^{ZZr} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\widetilde{\ell}_b^* \widetilde{\ell}_a ZZ} \right|^2 \frac{s^2 - 4m_Z^2 s + 12 m_Z^4}{8 m_Z^4} ; \tag{4.15}$$

- slepton $(\widetilde{\ell}_c)$ exchange:

$$\begin{aligned}
\widetilde{w}_{ZZ}^{(\widetilde{\ell})} = \frac{1}{m_Z^4} \sum_{c,d=1}^{2} & C^{\widetilde{\ell}_b^* \widetilde{\ell}_c Z} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a Z} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d Z *} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a Z *} \times \\
& \times \Bigg[ \mathcal{T}_4 - 2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 2m_Z^2 \right) \mathcal{T}_3 + \\
& + \left[ m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 + 4 m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 + 2 m_Z^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 3 m_Z^2 \right) \right] \mathcal{T}_2 - \\
& - 2 \Big[ \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) \left( m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 - m_Z^4 \right) + \\
& + m_Z^2 \left( m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 - 4 m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 + 2 m_Z^4 \right) \Big] \mathcal{T}_1 + \\
& + \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right)^2 \left( m_{\widetilde{\ell}_b}^2 - m_Z^2 \right)^2 \mathcal{T}_0 - \mathcal{Y}_4 + \\
& + \left[ s \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2 m_Z^2 \right) - 2 \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right) \left( m_{\widetilde{\ell}_b}^2 - m_Z^2 \right) \right] \mathcal{Y}_2 - \\
& - \Big[ s^2 \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right) \left( m_{\widetilde{\ell}_b}^2 - m_Z^2 \right) + \\
& + s \Big\{ - m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) + 3 m_Z^2 \left( m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 \right) - \\
& - 3 m_Z^4 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) + 2 m_Z^6 \Big\} + \\
& + \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right)^2 \left( m_{\widetilde{\ell}_b}^2 - m_Z^2 \right)^2 \Big] \mathcal{Y}_0 \Bigg] ;
\end{aligned} \tag{4.16}$$

- Higgs $(h, H)$ (+ Point) - slepton $(\widetilde{\ell}_c)$ interference:

$$\begin{aligned}
\widetilde{w}_{ZZ}^{(h,H,P-\widetilde{\ell})} = \frac{1}{2m_Z^4} \sum_{c=1}^{2} \mathrm{Re} & \left[ \left( \sum_{r=h,H} \frac{C^{ZZr} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\widetilde{\ell}_b^* \widetilde{\ell}_a ZZ} \right)^* C^{\widetilde{\ell}_b^* \widetilde{\ell}_c Z} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a Z} \right] \times \\
\times \Big[ & s^2 + s \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2 m_{\widetilde{\ell}_c}^2 - 4 m_Z^2 \right) + 2 m_Z^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2 m_{\widetilde{\ell}_c}^2 + 2 m_Z^2 \right) - \\
& - 2 \Big[ s \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2 \right) \left( m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2 \right) + \\
& + 2 m_Z^2 \Big\{ m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 - m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 + m_{\widetilde{\ell}_c}^2 \left( m_{\widetilde{\ell}_c}^2 - m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 - 2 m_Z^2 \right) - \\
& - m_Z^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - m_Z^2 \right) \Big\} \Big] \mathcal{F} \Big] .
\end{aligned} \tag{4.17}$$

### 4.3 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to Z\gamma$

This process proceeds via the four-point interaction, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to Z\gamma} = \widetilde{w}_{Z\gamma}^{(P)} + \widetilde{w}_{Z\gamma}^{(\widetilde{\ell})} + \widetilde{w}_{Z\gamma}^{(P-\widetilde{\ell})} \ : \tag{4.18}$$

- <u>Point interaction:</u>

$$\widetilde{w}_{Z\gamma}^{(P)} = 3\left|C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z\gamma}\right|^2 ; \tag{4.19}$$

- <u>slepton ($\widetilde{\ell}_c$) exchange:</u>

$$\begin{aligned}
\widetilde{w}_{Z\gamma}^{(\widetilde{\ell})} = \frac{2}{m_Z^2}\sum_{c,d=1}^{2} & C^{\widetilde{\ell}_c^*\widetilde{\ell}_a Z}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c \gamma}C^{\widetilde{\ell}_d^*\widetilde{\ell}_a Z*}C^{\widetilde{\ell}_b^*\widetilde{\ell}_d \gamma*} \times \\
&\times \Bigg[ - \mathcal{T}_3^t + \left(2m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 + 2m_Z^2\right)\mathcal{T}_2^t - \\
&\quad - \left\{\left(m_{\widetilde{\ell}_a}^2 - m_Z^2\right)^2 - 2m_{\widetilde{\ell}_b}^2\left(m_{\widetilde{\ell}_a}^2 + m_Z^2\right)\right\}\mathcal{T}_1^t - m_{\widetilde{\ell}_b}^2\left(m_{\widetilde{\ell}_a}^2 - m_Z^2\right)^2\mathcal{T}_0^t\Bigg] + \\
+ \frac{2}{m_Z^2}\sum_{c,d=1}^{2} & C^{\widetilde{\ell}_c^*\widetilde{\ell}_a \gamma}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c Z}C^{\widetilde{\ell}_d^*\widetilde{\ell}_a \gamma*}C^{\widetilde{\ell}_b^*\widetilde{\ell}_d Z*} \times \\
&\times \Bigg[ - \mathcal{T}_3^u + (2m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_a}^2 + 2m_Z^2)\mathcal{T}_2^u - \\
&\quad - \left\{\left(m_{\widetilde{\ell}_b}^2 - m_Z^2\right)^2 - 2m_{\widetilde{\ell}_a}^2\left(m_{\widetilde{\ell}_b}^2 + m_Z^2\right)\right\}\mathcal{T}_1^u - m_{\widetilde{\ell}_a}^2\left(m_{\widetilde{\ell}_b}^2 - m_Z^2\right)^2\mathcal{T}_0^u\Bigg] + \\
+ \frac{1}{m_Z^2}\mathrm{Re}\sum_{c,d=1}^{2} & C^{\widetilde{\ell}_c^*\widetilde{\ell}_a Z}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c \gamma}C^{\widetilde{\ell}_d^*\widetilde{\ell}_a \gamma*}C^{\widetilde{\ell}_b^*\widetilde{\ell}_d Z*}\left(s - 2m_{\widetilde{\ell}_a}^2 - 2m_{\widetilde{\ell}_b}^2 + m_Z^2\right) \times \\
&\times \Bigg[ - 2\mathcal{Y}_2 + \frac{1}{s}\left(s + m_Z^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)\mathcal{Y}_1 + \\
&\quad + \Bigg[s\left(m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2m_Z^2\right) - \left(m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4\right) + 3m_Z^2\left(m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2\right) - \\
&\quad - \frac{1}{s}m_Z^2\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)^2 - \frac{1}{2s^2}m_Z^4\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)^2\Bigg]\mathcal{Y}_0\Bigg] ;
\end{aligned} \tag{4.20}$$

- <u>Point - slepton ($\widetilde{\ell}_c$) interference:</u>

$$\begin{aligned}
\widetilde{w}_{Z\gamma}^{(P-\widetilde{\ell})} = \frac{1}{m_Z^2}\mathrm{Re}\sum_{c=1}^{2} & C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z\gamma*}C^{\widetilde{\ell}_c^*\widetilde{\ell}_a Z}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c \gamma} \times \\
&\times \Bigg[ - 3m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 2m_{\widetilde{\ell}_c}^2 - 2m_Z^2 - \frac{1}{s}m_Z^2\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right) + \\
&\quad + \Bigg[s\left(m_{\widetilde{\ell}_c}^2 - m_{\widetilde{\ell}_a}^2 + m_Z^2\right) + \left(2m_{\widetilde{\ell}_a}^2 - m_Z^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_Z^2\right) + \\
&\quad + m_{\widetilde{\ell}_c}^2(2m_{\widetilde{\ell}_c}^2 - 4m_{\widetilde{\ell}_a}^2 - 3m_Z^2) - 2m_Z^2 m_{\widetilde{\ell}_b}^2\Bigg]\mathcal{F}^t\Bigg] +
\end{aligned}$$

$$+\frac{1}{m_Z^2}\,\mathrm{Re}\sum_{c=1}^{2} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z\gamma *}C^{\widetilde{\ell}_c^*\widetilde{\ell}_a\gamma}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c Z}\times$$

$$\times\Bigg[-3m_{\widetilde{\ell}_b}^2+m_{\widetilde{\ell}_a}^2+2m_{\widetilde{\ell}_c}^2-2m_Z^2+\frac{1}{s}m_Z^2\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)+$$

$$+\Big[s\left(m_{\widetilde{\ell}_c}^2-m_{\widetilde{\ell}_b}^2+m_Z^2\right)+\left(2m_{\widetilde{\ell}_b}^2-m_Z^2\right)\left(m_{\widetilde{\ell}_b}^2-m_Z^2\right)+$$

$$+\,m_{\widetilde{\ell}_c}^2\left(2m_{\widetilde{\ell}_c}^2-4m_{\widetilde{\ell}_b}^2-3m_Z^2\right)-2m_Z^2m_{\widetilde{\ell}_a}^2\Big]\mathcal{F}^u\Bigg]. \qquad (4.21)$$

### 4.4 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma\gamma$

This process proceeds via the four-point interaction, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a=1,2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to\gamma\gamma}=\widetilde{w}_{\gamma\gamma}^{(P)}+\widetilde{w}_{\gamma\gamma}^{(\widetilde{\ell})}+\widetilde{w}_{\gamma\gamma}^{(P-\widetilde{\ell})}\;: \qquad (4.22)$$

- Point interaction:

$$\widetilde{w}_{\gamma\gamma}^{(P)}=8e^4\delta_{ab}\,; \qquad (4.23)$$

- slepton ($\widetilde{\ell}_a$) exchange:

$$\widetilde{w}_{\gamma\gamma}^{(\widetilde{\ell})}=e^4\delta_{ab}\left[4\left(\mathcal{T}_2+2m_{\widetilde{\ell}_a}^2\mathcal{T}_1+m_{\widetilde{\ell}_a}^4\mathcal{T}_0\right)-\left(s-4m_{\widetilde{\ell}_a}^2\right)^2\mathcal{Y}_0\right]; \qquad (4.24)$$

- Point - slepton ($\widetilde{\ell}_c$) interference:

$$\widetilde{w}_{\gamma\gamma}^{(P-\widetilde{\ell})}=2e^4\delta_{ab}\left[-4+\left(s-8m_{\widetilde{\ell}_a}^2\right)\mathcal{F}\right]. \qquad (4.25)$$

Next, there are processes of the type $\widetilde{\ell}_a\widetilde{\ell}_b^*\to$ vector boson - Higgs boson: $W^\pm H^\mp$, $Zh$, $ZH$, $ZA$, $\gamma h$, $\gamma H$ and $\gamma A$.

### 4.5 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to W^\pm H^\mp$

The process $\widetilde{\ell}_a\widetilde{\ell}_b^*\to W^+H^-$ involves the $s$-channel CP-even Higgs boson ($h$ and $H$) and CP-odd Higgs boson ($A$) exchange, and the $u$-channel sneutrino ($\widetilde{\nu}_\ell$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to W^+H^-}=\widetilde{w}_{W^+H^-}^{(h,H,A)}+\widetilde{w}_{W^+H^-}^{(\widetilde{\nu})}+\widetilde{w}_{W^+H^-}^{(h,H,A-\widetilde{\nu})}\;: \qquad (4.26)$$

- Higgs $(h,H,A)$ exchange:

$$\widetilde{w}_{W^+H^-}^{(h,H,A)}=\left|\sum_{r=h,H,A}\frac{C^{W^-H^+r}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s-m_r^2+i\Gamma_r m_r}\right|^2\frac{s^2-2s(m_{H^\pm}^2+m_W^2)+(m_{H^\pm}^2-m_W^2)^2}{m_W^2}\,; \qquad (4.27)$$

- sneutrino ($\widetilde{\nu}_\ell$) exchange:

$$\widetilde{w}_{W^+H^-}^{(\widetilde{\nu})}=\frac{1}{m_W^2}\left|C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a W^+}C^{\widetilde{\ell}_b^*\widetilde{\nu}_\ell H^-}\right|^2\times$$

$$\times\left[\mathcal{T}_2^u-2\left(m_{\widetilde{\ell}_b}^2+m_W^2\right)\mathcal{T}_1^u+\left(m_{\widetilde{\ell}_b}^2-m_W^2\right)^2\mathcal{T}_0^u\right]; \qquad (4.28)$$

- Higgs $(h,H,A)$ - sneutrino $(\widetilde{\nu}_\ell)$ interference:

$$\widetilde{w}^{(h,H,A-\widetilde{\nu})}_{W^+H^-} = -\frac{2}{m_W^2}\,\mathrm{Re}\left[\left(\sum_{r=h,H,A}\frac{C^{W^-H^+r}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s-m_r^2+i\Gamma_r m_r}\right)^* C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a W^+}C^{\widetilde{\ell}_b^*\widetilde{\nu}_\ell H^-}\right]\times$$
$$\times\left[s-m_{H^\pm}^2+m_W^2+\left\{s\left(m_{\widetilde{\nu}_\ell}^2+m_W^2-m_{\widetilde{\ell}_b}^2\right)+m_W^2\left(m_{\widetilde{\ell}_b}^2-2m_{\widetilde{\ell}_a}^2+m_{\widetilde{\nu}_\ell}^2-m_W^2\right)+\right.\right.$$
$$\left.\left.+\,m_{H^\pm}^2\left(m_{\widetilde{\ell}_b}^2-m_{\widetilde{\nu}_\ell}^2+m_W^2\right)\right\}\mathcal{F}^u\right]. \qquad (4.29)$$

The contribution from $\widetilde{\ell}_a\widetilde{\ell}_b^* \to W^-H^+$ can be obtained by interchanging the indices $a$ and $b$, but this is obviously equal to the one above:

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to W^-H^+} = \widetilde{w}_{\widetilde{\ell}_b\widetilde{\ell}_a^*\to W^+H^-}\,. \qquad (4.30)$$

### 4.6 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to Zh, ZH$

The process $\widetilde{\ell}_a\widetilde{\ell}_b^* \to Zh$ proceeds via the $s$-channel CP-odd Higgs boson $(A)$ exchange, the $s$-channel Z-boson exchange, and the $t$- and $u$-channel slepton $(\widetilde{\ell}_a,\ a=1,2)$ exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to Zh} = \widetilde{w}^{(A)}_{Zh}+\widetilde{w}^{(Z)}_{Zh}+\widetilde{w}^{(\widetilde{\ell})}_{Zh}+\widetilde{w}^{(A-Z)}_{Zh}+\widetilde{w}^{(A-\widetilde{\ell})}_{Zh}+\widetilde{w}^{(Z-\widetilde{\ell})}_{Zh}\ : \qquad (4.31)$$

- $A$ exchange:

$$\widetilde{w}^{(A)}_{Zh} = \left|\frac{C^{ZhA}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a A}}{s-m_A^2+i\Gamma_A m_A}\right|^2 \frac{s^2-2s(m_h^2+m_Z^2)+(m_h^2-m_Z^2)^2}{m_Z^2}\,; \qquad (4.32)$$

- $Z$ exchange:

$$\widetilde{w}^{(Z)}_{Zh} = \frac{1}{12m_Z^6}\left|\frac{C^{ZZh}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right|^2\times$$
$$\times\left[s^2\left\{3\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2+m_Z^4\right\}-\right.$$
$$-\,2s\left\{3\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2(m_h^2+2m_Z^2)+m_Z^4\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2+m_h^2-5m_Z^2\right)\right\}+$$
$$+\,m_Z^4\left\{(m_h^2-m_Z^2)^2+4\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)(m_h^2-5m_Z^2)\right\}+$$
$$+\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2(3m_h^4+6m_Z^2m_h^2+19m_Z^4)-$$
$$-\,\frac{2}{s}m_Z^2\left\{\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2(3m_h^4-2m_Z^2m_h^2+m_Z^4)+\right.$$
$$\left.+\,m_Z^2(m_h^2-m_Z^2)^2\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)\right\}+$$
$$\left.+\,\frac{4}{s^2}m_Z^4(m_h^2-m_Z^2)^2\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2\right]; \qquad (4.33)$$

• slepton ($\widetilde{\ell}_c$) exchange:

$$
\begin{aligned}
\widetilde{w}_{Zh}^{(\widetilde{\ell})} = & \frac{1}{m_Z^2} \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a Z} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a Z*} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d h*} \times \\
& \qquad \times \left[ \mathcal{T}_2^t - 2 \left( m_{\widetilde{\ell}_a}^2 + m_Z^2 \right) \mathcal{T}_1^t + \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right)^2 \mathcal{T}_0^t \right] + \\
& + \frac{1}{m_Z^2} \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c Z} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a h*} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d Z*} \times \\
& \qquad \times \left[ \mathcal{T}_2^u - 2 \left( m_{\widetilde{\ell}_b}^2 + m_Z^2 \right) \mathcal{T}_1^u + \left( m_{\widetilde{\ell}_b}^2 - m_Z^2 \right)^2 \mathcal{T}_0^u \right] + \\
& + \frac{1}{m_Z^2} \operatorname{Re} \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a Z} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a h*} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d Z*} \times \\
& \qquad \times \Bigg[ - 2\mathcal{Y}_2 + \frac{1}{s} \left( s - m_h^2 + m_Z^2 \right) \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right) \mathcal{Y}_1 + \\
& \qquad + \Big\{ s \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 - 2m_Z^2 \right) - \left( m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_b}^4 \right) - 2m_Z^2 m_h^2 + \\
& \qquad + \left( 3m_Z^2 - m_h^2 \right) \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) + \frac{1}{s} \left( m_h^2 - m_Z^2 \right) \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right)^2 - \\
& \qquad - \frac{1}{2s^2} \left( m_h^2 - m_Z^2 \right)^2 \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right)^2 \Big\} \mathcal{Y}_0 \Bigg];
\end{aligned}
\tag{4.34}
$$

• $A$ - $Z$ interference:

$$
\begin{aligned}
\widetilde{w}_{Zh}^{(A-Z)} = & -\frac{1}{s m_Z^4} \operatorname{Re} \left[ \left( \frac{C^{ZhA} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A} \right)^* \left( \frac{C^{ZZh} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} \right) \right] \times \\
& \times \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right) \left( s - m_Z^2 \right) \left[ s^2 - 2s \left( m_h^2 + m_Z^2 \right) + \left( m_h^2 - m_Z^2 \right)^2 \right];
\end{aligned}
\tag{4.35}
$$

• Higgs ($A$) - slepton ($\widetilde{\ell}_c$) interference:

$$
\begin{aligned}
\widetilde{w}_{Zh}^{(A-\widetilde{\ell})} = & -\frac{2}{m_Z^2} \operatorname{Re} \sum_{c=1}^{2} \left( \frac{C^{ZhA} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A} \right)^* \times \\
& \times \Bigg[ C^{\widetilde{\ell}_c^* \widetilde{\ell}_a Z} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} \Big\{ - \left( s - m_h^2 + m_Z^2 \right) + \\
& \qquad + \left[ s \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2 \right) - m_{\widetilde{\ell}_a}^2 \left( m_h^2 + m_Z^2 \right) + \right. \\
& \qquad \left. + \left( m_{\widetilde{\ell}_c}^2 - m_Z^2 \right) \left( m_h^2 - m_Z^2 \right) + 2 m_Z^2 m_{\widetilde{\ell}_b}^2 \right] \mathcal{F}^t \Big\} -
\end{aligned}
$$

$$
\begin{aligned}
&- C^{\widetilde{\ell}_c^*\widetilde{\ell}_a h} C^{\widetilde{\ell}_b^*\widetilde{\ell}_c Z}\Big\{ -\left(s - m_h^2 + m_Z^2\right) + \\
&\qquad + \Big[ s\left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2\right) - m_{\widetilde{\ell}_b}^2\left(m_h^2 + m_Z^2\right) + \\
&\qquad + \left(m_{\widetilde{\ell}_c}^2 - m_Z^2\right)\left(m_h^2 - m_Z^2\right) + 2m_Z^2 m_{\widetilde{\ell}_a}^2 \Big] \mathcal{F}^u \Big\}\Bigg] ;
\end{aligned}
\tag{4.36}
$$

- $Z$ - slepton ($\widetilde{\ell}_c$) interference:

$$
\begin{aligned}
\widetilde{w}_{Zh}^{(Z-\widetilde{\ell})} = \frac{1}{m_Z^4}\,\mathrm{Re}\sum_{c=1}^{2} & \left(\frac{C^{ZZh}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right)^* \times \\
\times \Bigg[ & C^{\widetilde{\ell}_c^*\widetilde{\ell}_a Z} C^{\widetilde{\ell}_b^*\widetilde{\ell}_c h}\Big\{ -\left(s - m_h^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right) + \\
&\quad + 2m_Z^2\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_c}^2 + m_Z^2\right) - \\
&\quad - \frac{1}{s} m_Z^2\left(m_h^2 - m_Z^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right) + \\
&\quad + \Big[ s\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 + m_Z^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2\right) + \\
&\qquad + m_Z^2\Big\{ -2m_{\widetilde{\ell}_c}^4 + m_{\widetilde{\ell}_c}^2\left(3m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 3m_Z^2\right) - \\
&\qquad\quad - 3m_{\widetilde{\ell}_a}^4 + 3m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 - 2m_{\widetilde{\ell}_b}^4 + \\
&\qquad\quad + m_Z^2\left(4m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2\right) - m_Z^4 \Big\} - \\
&\qquad - m_h^2\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 + m_Z^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_c}^2 + m_Z^2\right)\Big]\mathcal{F}^t \Big\} + \\
& + C^{\widetilde{\ell}_c^*\widetilde{\ell}_a h} C^{\widetilde{\ell}_b^*\widetilde{\ell}_c Z}\Big\{ \left(s - m_h^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right) + 2m_Z^2\left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_c}^2 + m_Z^2\right) + \\
&\qquad + \frac{1}{s} m_Z^2\left(m_h^2 - m_Z^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right) + \\
&\qquad + \Big[ -s\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 - m_Z^2\right)\left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2\right) + \\
&\qquad\quad + m_Z^2\Big\{ -2m_{\widetilde{\ell}_c}^4 + m_{\widetilde{\ell}_c}^2\left(3m_{\widetilde{\ell}_b}^2 + m_{\widetilde{\ell}_a}^2 + 3m_Z^2\right) - \\
&\qquad\qquad - 3m_{\widetilde{\ell}_b}^4 + 3m_{\widetilde{\ell}_a}^2 m_{\widetilde{\ell}_b}^2 - 2m_{\widetilde{\ell}_a}^4 + \\
&\qquad\qquad + m_Z^2\left(4m_{\widetilde{\ell}_b}^2 + m_{\widetilde{\ell}_a}^2\right) - m_Z^4 \Big\} + \\
&\qquad\quad + m_h^2\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 - m_Z^2\right) \times \\
&\qquad\quad \times \left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_c}^2 + m_Z^2\right)\Big]\mathcal{F}^u \Big\}\Bigg] .
\end{aligned}
\tag{4.37}
$$

The expression for $\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to ZH}$ can be obtained by a simple substitution $h \to H$.

### 4.7 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to ZA$

This process proceeds via the $s$-channel CP-even Higgs boson ($h$ and $H$) exchange, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a=1,2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to ZA} = \widetilde{w}_{ZA}^{(h,H)} + \widetilde{w}_{ZA}^{(\widetilde{\ell})} + \widetilde{w}_{ZA}^{(h,H-\widetilde{\ell})} \quad : \tag{4.38}$$

• Higgs $(h,H)$ exchange:

$$\widetilde{w}_{ZA}^{(h,H)} = \left| \sum_{r=h,H} \frac{C^{ZAr} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} \right|^2 \frac{s^2 - 2s(m_A^2+m_Z^2) + (m_A^2 - m_Z^2)^2}{m_Z^2} \, ; \tag{4.39}$$

The expression for $\widetilde{w}_{ZA}^{(\widetilde{\ell})}$ can be obtained from $\widetilde{w}_{Zh}^{(\widetilde{\ell})}$ by a simple substitution $h \to A$.

• Higgs $(h,H)$ - slepton $(\widetilde{\ell}_c)$ interference:

$$\begin{aligned}
\widetilde{w}_{ZA}^{(h,H-\widetilde{\ell})} = \frac{2}{m_Z^2} \,\mathrm{Re} \sum_{c=1}^{2} &\left( \sum_{r=h,H} \frac{C^{ZAr} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} \right)^* \times \\
&\times \Bigg[ C^{\widetilde{\ell}_c^*\widetilde{\ell}_a Z} C^{\widetilde{\ell}_b^*\widetilde{\ell}_c A} \Bigg\{ -(s - m_A^2 + m_Z^2) + \\
&\qquad + \Big[ s\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2\right) - m_{\widetilde{\ell}_a}^2 (m_A^2 + m_Z^2) + \\
&\qquad + \left(m_{\widetilde{\ell}_c}^2 - m_Z^2\right)(m_A^2 - m_Z^2) + 2 m_Z^2 m_{\widetilde{\ell}_b}^2 \Big] \mathcal{F}^t \Bigg\} - \\
&- C^{\widetilde{\ell}_c^*\widetilde{\ell}_a A} C^{\widetilde{\ell}_b^*\widetilde{\ell}_c Z} \Bigg\{ -(s - m_A^2 + m_Z^2) + \\
&\qquad + \Big[ s\left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_c}^2 - m_Z^2\right) - m_{\widetilde{\ell}_b}^2 (m_A^2 + m_Z^2) + \\
&\qquad + \left(m_{\widetilde{\ell}_c}^2 - m_Z^2\right)(m_A^2 - m_Z^2) + 2 m_Z^2 m_{\widetilde{\ell}_a}^2 \Big] \mathcal{F}^u \Bigg\} \Bigg] .
\end{aligned} \tag{4.40}$$

### 4.8 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma h, \gamma H$

The process $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma h$ proceeds only via the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a=1,2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to\gamma h} = \widetilde{w}_{\gamma h}^{(\widetilde{\ell})} \quad : \tag{4.41}$$

• slepton $(\widetilde{\ell}_c)$ exchange:

$$\begin{aligned}
\widetilde{w}_{\gamma h}^{(\widetilde{\ell})} = -2e^2 \left| C^{\widetilde{\ell}_b^*\widetilde{\ell}_a h} \right|^2 &\Big[ \left( \mathcal{T}_1^t + m_{\widetilde{\ell}_a}^2 \mathcal{T}_0^t \right) + \left( \mathcal{T}_1^u + m_{\widetilde{\ell}_b}^2 \mathcal{T}_0^u \right) + \\
&+ \left( s + m_h^2 - 2m_{\widetilde{\ell}_a}^2 - 2m_{\widetilde{\ell}_b}^2 \right) \mathcal{Y}_0 \Big] .
\end{aligned} \tag{4.42}$$

The expression for $\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to\gamma H}$ can be obtained by a simple substitution $h \to H$.

### 4.9 $\widetilde{\ell}_a \widetilde{\ell}_b^* \to \gamma A$

This process proceeds only via the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a \widetilde{\ell}_b^* \to \gamma A} = \widetilde{w}_{\gamma A}^{(\widetilde{\ell})} \; : \tag{4.43}$$

$\widetilde{w}_{\gamma A}^{(\widetilde{\ell})}$ can be obtained from $\widetilde{w}_{\gamma h}^{(\widetilde{\ell})}$ by a simple substitution $h \to A$.

Next we proceed to present the results for $\widetilde{\ell}_a \widetilde{\ell}_b^* \to$ Higgs-Higgs pairs: $hh$, $HH$, $hH$, $hA$, $HA$, $AA$ and $H^+H^-$.

### 4.10 $\widetilde{\ell}_a \widetilde{\ell}_b^* \to hh, HH, hH$

The process $\widetilde{\ell}_a \widetilde{\ell}_b^* \to hH$ proceeds via the $s$-channel CP-even Higgs boson ($h$ and $H$) exchange, the four-point interaction, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a \widetilde{\ell}_b^* \to hH} = \widetilde{w}_{hH}^{(h,H,P)} + \widetilde{w}_{hH}^{(\widetilde{\ell})} + \widetilde{w}_{hH}^{(h,H,P-\widetilde{\ell})} \; : \tag{4.44}$$

- Higgs $(h, H)$ exchange (+ Point interaction):

$$\widetilde{w}_{hH}^{(h,H,P)} = \left| \sum_{r=h,H} \frac{C^{hHr} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\widetilde{\ell}_b^* \widetilde{\ell}_a hH} \right|^2 ; \tag{4.45}$$

- slepton ($\widetilde{\ell}_c$) exchange:

$$\begin{aligned} \widetilde{w}_{hH}^{(\widetilde{\ell})} = & \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c H} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a h *} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d H *} \mathcal{T}_0^t + \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a H} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a H *} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d h *} \mathcal{T}_0^u - \\ & - 2\,\mathrm{Re} \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c H} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a H *} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d h *} \mathcal{Y}_0 \, ; \end{aligned} \tag{4.46}$$

- Higgs $(h, H)$ (+ Point) - slepton ($\widetilde{\ell}_c$) interference:

$$\begin{aligned} \widetilde{w}_{hH}^{(h,H,P-\widetilde{\ell})} = & \, 2\,\mathrm{Re} \sum_{c=1}^{2} \left( \sum_{r=h,H} \frac{C^{hHr} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\widetilde{\ell}_b^* \widetilde{\ell}_a hH} \right)^* \times \\ & \times \left[ C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c H} \mathcal{F}^t + C^{\widetilde{\ell}_c^* \widetilde{\ell}_a H} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} \mathcal{F}^u \right] . \end{aligned} \tag{4.47}$$

The expressions for $hh$ final state are obtained from the above by replacing $C^{hHr}$, $C^{\widetilde{\ell}_b^* \widetilde{\ell}_c H}$, $C^{\widetilde{\ell}_c^* \widetilde{\ell}_a H}$, $C^{hH \widetilde{\ell}_b^* \widetilde{\ell}_a}$ with $C^{hhr}$, $C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h}$, $C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h}$, $C^{hh \widetilde{\ell}_b^* \widetilde{\ell}_a}$, respectively, and multiplying $\widetilde{w}$ by a factor of 1/2 for identical particles in the final state. The contributions for $HH$ final state are obtained in an analogous way.

### 4.11 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to hA, HA$

The process $\widetilde{\ell}_a\widetilde{\ell}_b^* \to hA$ proceeds via the $s$-channel CP-odd Higgs boson ($A$) exchange, the $s$-channel Z-boson exchange, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^* \to hA} = \widetilde{w}_{hA}^{(A)} + \widetilde{w}_{hA}^{(Z)} + \widetilde{w}_{hA}^{(\widetilde{\ell})} + \widetilde{w}_{hA}^{(A-Z)} + \widetilde{w}_{hA}^{(A-\widetilde{\ell})} + \widetilde{w}_{hA}^{(Z-\widetilde{\ell})} : \tag{4.48}$$

- <u>$A$ exchange:</u>

$$\widetilde{w}_{hA}^{(A)} = \left| \frac{C^{hAA} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A} \right|^2 ; \tag{4.49}$$

- <u>$Z$ exchange:</u>

$$\begin{aligned} \widetilde{w}_{hA}^{(Z)} = & \frac{1}{3s^2 m_Z^4} \left| \frac{C^{hAZ} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \times \\ & \times \Bigg[ s^4 m_Z^4 - 2s^3 m_Z^4 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + m_A^2 + m_h^2 \right) + \\ & + s^2 \Bigg\{ \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right)^2 \left[ 3 \left( m_A^2 - m_h^2 \right)^2 + m_Z^4 \right] + \\ & \qquad + 4 m_Z^4 \left( m_A^2 + m_h^2 \right) \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) + m_Z^4 \left( m_A^2 - m_h^2 \right)^2 \Bigg\} - \\ & - 2s m_Z^2 \Bigg\{ \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right)^2 \left[ 3 \left( m_A^2 - m_h^2 \right)^2 + m_Z^2 \left( m_A^2 + m_h^2 \right) \right] + \\ & \qquad + m_Z^2 \left( m_A^2 - m_h^2 \right)^2 \left( m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 \right) \Bigg\} + \\ & + 4 m_Z^4 \left( m_A^2 - m_h^2 \right)^2 \left( m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right)^2 \Bigg] ; \end{aligned} \tag{4.50}$$

- <u>slepton ($\widetilde{\ell}_c$) exchange:</u>

$$\begin{aligned} \widetilde{w}_{hA}^{(\widetilde{\ell})} = & \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c A} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a h *} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d A *} \mathcal{T}_0^t + \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a A} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a A *} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d h *} \mathcal{T}_0^u - \\ & - 2\,\mathrm{Re} \sum_{c,d=1}^{2} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c A} C^{\widetilde{\ell}_d^* \widetilde{\ell}_a A *} C^{\widetilde{\ell}_b^* \widetilde{\ell}_d h *} \mathcal{Y}_0 ; \end{aligned} \tag{4.51}$$

- <u>$A$ - $Z$ interference:</u>

$$\begin{aligned} \widetilde{w}_{hA}^{(A-Z)} = & 2\,\mathrm{Re} \left[ \left( \frac{C^{hAA} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A} \right)^* \left( \frac{C^{hAZ} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} \right) \right] \times \\ & \times \frac{(m_A^2 - m_h^2)(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2)(s - m_Z^2)}{s m_Z^2} ; \end{aligned} \tag{4.52}$$

- <u>Higgs ($A$) - slepton ($\widetilde{\ell}_c$) interference:</u>

$$\widetilde{w}_{hA}^{(A-\widetilde{\ell})} = 2\,\mathrm{Re} \sum_{c=1}^{2} \left( \frac{C^{hAA} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A} \right)^* \left[ C^{\widetilde{\ell}_c^* \widetilde{\ell}_a h} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c A} \mathcal{F}^t + C^{\widetilde{\ell}_c^* \widetilde{\ell}_a A} C^{\widetilde{\ell}_b^* \widetilde{\ell}_c h} \mathcal{F}^u \right] ; \tag{4.53}$$

- <u>$Z$ - slepton ($\widetilde{\ell}_c$) interference:</u>

$$\begin{aligned}
\widetilde{w}_{hA}^{(Z-\widetilde{\ell})} = &-\frac{2}{m_Z^2}\sum_{c=1}^{2}\mathrm{Re}\left[\left(\frac{C^{hAZ}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right)^* C^{\widetilde{\ell}_c^*\widetilde{\ell}_a h}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c A}\right]\times \\
&\times\left[2m_Z^2-\left\{\left(m_A^2-m_h^2\right)\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)+m_Z^2\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)+\right.\right. \\
&\left.\left.+\,m_Z^2\left(-s-2m_{\widetilde{\ell}_c}^2+m_A^2+m_h^2\right)\right\}\mathcal{F}^t\right]+ \\
&+\frac{2}{m_Z^2}\sum_{c=1}^{2}\mathrm{Re}\left[\left(\frac{C^{hAZ}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right)^* C^{\widetilde{\ell}_c^*\widetilde{\ell}_a A}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c h}\right]\times \\
&\times\left[2m_Z^2-\left\{-\left(m_A^2-m_h^2\right)\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)+m_Z^2\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2\right)+\right.\right. \\
&\left.\left.+\,m_Z^2\left(-s-2m_{\widetilde{\ell}_c}^2+m_A^2+m_h^2\right)\right\}\mathcal{F}^u\right]. \qquad (4.54)
\end{aligned}$$

The expression for $\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to HA}$ can be obtained by a simple substitution $h\to H$.

### 4.12 $\widetilde{\ell}_a\widetilde{\ell}_b^*\to AA$

This process proceeds via the $s$-channel CP-even Higgs boson ($h$ and $H$) exchange, the four-point interaction, and the $t$- and $u$-channel slepton ($\widetilde{\ell}_a$, $a=1,2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to AA}=\widetilde{w}_{AA}^{(h,H,P)}+\widetilde{w}_{AA}^{(\widetilde{\ell})}+\widetilde{w}_{AA}^{(h,H,P-\widetilde{\ell})}\ : \qquad (4.55)$$

- <u>Higgs $(h,H)$ exchange (+ Point interaction):</u>

$$\widetilde{w}_{AA}^{(h,H,P)}=\frac{1}{2}\left|\sum_{r=h,H}\frac{C^{AAr}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s-m_r^2+i\Gamma_r m_r}-C^{\widetilde{\ell}_b^*\widetilde{\ell}_a AA}\right|^2; \qquad (4.56)$$

- <u>slepton ($\widetilde{\ell}_c$) exchange:</u>

$$\widetilde{w}_{AA}^{(\widetilde{\ell})}=\sum_{c,d=1}^{2}C^{\widetilde{\ell}_c^*\widetilde{\ell}_a A}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c A}C^{\widetilde{\ell}_d^*\widetilde{\ell}_a A*}C^{\widetilde{\ell}_b^*\widetilde{\ell}_d A*}(\mathcal{T}_0-\mathcal{Y}_0)\,; \qquad (4.57)$$

- <u>Higgs $(h,H)$ (+ Point) - slepton ($\widetilde{\ell}_c$) interference:</u>

$$\widetilde{w}_{AA}^{(h,H,P-\widetilde{\ell})}=2\sum_{c=1}^{2}\mathrm{Re}\left[\left(\sum_{r=h,H}\frac{C^{AAr}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s-m_r^2+i\Gamma_r m_r}-C^{\widetilde{\ell}_b^*\widetilde{\ell}_a AA}\right)^* C^{\widetilde{\ell}_c^*\widetilde{\ell}_a A}C^{\widetilde{\ell}_b^*\widetilde{\ell}_c A}\right]\mathcal{F}\,. \qquad (4.58)$$

### 4.13 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to H^+H^-$

This process proceeds via the $s$-channel CP-even Higgs boson ($h$ and $H$) exchange, the four-point interaction, the $s$-channel Z-boson and photon exchange, and the $u$-channel sneutrino ($\widetilde{\nu}_\ell$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to H^+H^-} = \widetilde{w}^{(h,H,P)}_{H^+H^-} + \widetilde{w}^{(Z,\gamma)}_{H^+H^-} + \widetilde{w}^{(\widetilde{\nu})}_{H^+H^-} + \widetilde{w}^{(h,H,P-\widetilde{\nu})}_{H^+H^-} + \widetilde{w}^{(Z,\gamma-\widetilde{\nu})}_{H^+H^-} : \tag{4.59}$$

- Higgs $(h, H)$ exchange (+ Point interaction):

$$\widetilde{w}^{(h,H,P)}_{H^+H^-} = \left| \sum_{r=h,H} \frac{C^{H^+H^-r}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s-m_r^2+i\Gamma_r m_r} - C^{\widetilde{\ell}_b^*\widetilde{\ell}_a H^+H^-} \right|^2 ; \tag{4.60}$$

- $Z, \gamma$ exchange:

$$\begin{aligned}\widetilde{w}^{(Z,\gamma)}_{H^+H^-} &= \frac{1}{3s}\left| \frac{C^{H^+H^-Z}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z} + \frac{e^2\delta_{ab}}{s} \right|^2 \times \\ &\times \left(s-4m^2_{H^\pm}\right)\left[s-\left(m_{\widetilde{\ell}_a}+m_{\widetilde{\ell}_b}\right)^2\right]\left[s-\left(m_{\widetilde{\ell}_a}-m_{\widetilde{\ell}_b}\right)^2\right] ;\end{aligned} \tag{4.61}$$

- sneutrino $(\widetilde{\nu}_\ell)$ exchange:

$$\widetilde{w}^{(\widetilde{\nu})}_{H^+H^-} = \left| C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a H^+}C^{\widetilde{\ell}_b^*\widetilde{\nu}_\ell H^-} \right|^2 \mathcal{T}_0 ; \tag{4.62}$$

- Higgs $(h, H)$ (+ Point) - sneutrino $(\widetilde{\nu}_\ell)$ interference:

$$\begin{aligned}&\widetilde{w}^{(h,H,P-\widetilde{\nu})}_{H^+H^-} = \\ &= 2\,\mathrm{Re}\left[\left(\sum_{r=h,H} \frac{C^{H^+H^-r}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s-m_r^2+i\Gamma_r m_r} - C^{\widetilde{\ell}_b^*\widetilde{\ell}_a H^+H^-}\right)^* C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a H^+}C^{\widetilde{\ell}_b^*\widetilde{\nu}_\ell H^-}\right]\mathcal{F} ;\end{aligned} \tag{4.63}$$

- $Z, \gamma$ - sneutrino $(\widetilde{\nu}_\ell)$ interference:

$$\begin{aligned}\widetilde{w}^{(Z,\gamma-\widetilde{\nu})}_{H^+H^-} &= 2\,\mathrm{Re}\left[\left(\frac{C^{H^+H^-Z}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z} + \frac{e^2\delta_{ab}}{s}\right)^* C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a H^+}C^{\widetilde{\ell}_b^*\widetilde{\nu}_\ell H^-}\right] \times \\ &\times \left[2+\left(s+2m^2_{\widetilde{\nu}_\ell}-2m^2_{H^\pm}-m^2_{\widetilde{\ell}_a}-m^2_{\widetilde{\ell}_b}\right)\mathcal{F}\right] .\end{aligned} \tag{4.64}$$

Finally, we present the results for $\widetilde{\ell}_a\widetilde{\ell}_b^* \to f\bar{f}$ where $f$ denotes any of the SM fermions.

### 4.14 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \ell\bar{\ell}$

This process involves the $s$-channel CP-even Higgs boson ($h$ and $H$) and CP-odd Higgs boson ($A$) exchange, the $s$-channel Z-boson and photon exchange, and the $t$-channel neutralino ($\chi_i^0$, $i = 1, 2, 3, 4$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to\ell\bar{\ell}} = \widetilde{w}^{(h,H,A)}_{\ell\bar{\ell}} + \widetilde{w}^{(Z,\gamma)}_{\ell\bar{\ell}} + \widetilde{w}^{(\chi^0)}_{\ell\bar{\ell}} + \widetilde{w}^{(A-Z)}_{\ell\bar{\ell}} + \widetilde{w}^{(h,H,A-\chi^0)}_{\ell\bar{\ell}} + \widetilde{w}^{(Z,\gamma-\chi^0)}_{\ell\bar{\ell}} : \tag{4.65}$$

• Higgs $(h, H, A)$ exchange:

$$\widetilde{w}_{\ell\bar{\ell}}^{(h,H,A)} = 2\left|\sum_{r=h,H} \frac{C_S^{\ell\ell r} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r}\right|^2 \left(s - 4m_\ell^2\right) + 2\left|\frac{C_P^{\ell\ell A} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A}\right|^2 s\,; \qquad (4.66)$$

• $Z, \gamma$ exchange:

$$\begin{aligned}
\widetilde{w}_{\ell\bar{\ell}}^{(Z,\gamma)} &= \frac{4}{3}\left|\frac{C_V^{\ell\ell Z} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} - \frac{e e_\ell \delta_{ab}}{s}\right|^2 \times \\
&\quad \times \left(s + 2m_\ell^2\right)\left\{s - \left(m_{\widetilde{\ell}_a} + m_{\widetilde{\ell}_b}\right)^2\right\}\left\{1 - \frac{(m_{\widetilde{\ell}_a} - m_{\widetilde{\ell}_b})^2}{s}\right\} + \\
&\quad + \frac{4}{3sm_Z^4}\left|\frac{C_A^{\ell\ell Z} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right|^2 \times \\
&\quad \times \left[s^3 m_Z^4 - 2s^2\left\{m_Z^4\left(m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2 + 2m_\ell^2\right) - 3m_\ell^2\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)^2\right\} + \right. \\
&\qquad + sm_Z^2\left\{\left(m_Z^2 - 12m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)^2 + 8m_\ell^2 m_Z^2\left(m_{\widetilde{\ell}_a}^2 + m_{\widetilde{\ell}_b}^2\right)\right\} + \\
&\qquad \left. + 2m_\ell^2 m_Z^4\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)^2\right], \qquad (4.67)
\end{aligned}$$

where $e_\ell = -e < 0$ is the electric charge for the charged lepton $\ell$ in the final state. For other fermion pair production, this should be replaced with the electric charge of the final particle. • neutralino $(\chi_i^0)$ exchange:

$$\begin{aligned}
\widetilde{w}_{\ell\bar{\ell}}^{(\chi^0)} &= \sum_{i,j=1}^{4}\left[C_{LLLL}^{abij}\left\{-\mathcal{T}_2 - \left(s - m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2\right)\mathcal{T}_1 - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}_b}^2 - m_\ell^2\right)\mathcal{T}_0\right\} - \right. \\
&\quad - 2m_\ell^2\left[C_{LRLR}^{abij} m_{\chi_i^0} m_{\chi_j^0}\mathcal{T}_0 + C_{LLRR}^{abij}\mathcal{T}_1\right] + C_{LRRL}^{abij} m_{\chi_i^0} m_{\chi_j^0}\left(s - 2m_\ell^2\right)\mathcal{T}_0 - \\
&\quad - m_\ell m_{\chi_i^0}\left[C_{LRLL}^{abij}\left\{\mathcal{T}_1 - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right)\mathcal{T}_0\right\} + \right. \\
&\qquad\qquad \left. + C_{RLLL}^{abij}\left\{\mathcal{T}_1 - \left(m_{\widetilde{\ell}_b}^2 - m_\ell^2\right)\mathcal{T}_0\right\}\right] - \qquad (4.68) \\
&\quad \left. - m_\ell m_{\chi_j^0}\left[C_{LLRL}^{abij}\left\{\mathcal{T}_1 - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right)\mathcal{T}_0\right\} + C_{LLLR}^{abij}\left\{\mathcal{T}_1 - \left(m_{\widetilde{\ell}_b}^2 - m_\ell^2\right)\mathcal{T}_0\right\}\right]\right],
\end{aligned}$$

where

$$\begin{aligned}
C_{LLLL}^{abij} &= C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell *} C_L^{\chi_i^0 \widetilde{\ell}_b^* \ell} C_L^{\chi_j^0 \widetilde{\ell}_b^* \ell *} C_L^{\chi_j^0 \widetilde{\ell}_a^* \ell} + (L \longrightarrow R)\,, \\
C_{LRLR}^{abij} &= C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell *} C_R^{\chi_i^0 \widetilde{\ell}_b^* \ell} C_L^{\chi_j^0 \widetilde{\ell}_b^* \ell *} C_R^{\chi_j^0 \widetilde{\ell}_a^* \ell} + (L \longleftrightarrow R)\,, \\
C_{LLRR}^{abij} &= C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell *} C_L^{\chi_i^0 \widetilde{\ell}_b^* \ell} C_R^{\chi_j^0 \widetilde{\ell}_b^* \ell *} C_R^{\chi_j^0 \widetilde{\ell}_a^* \ell} + (L \longleftrightarrow R)\,, \\
C_{LRRL}^{abij} &= C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell *} C_R^{\chi_i^0 \widetilde{\ell}_b^* \ell} C_R^{\chi_j^0 \widetilde{\ell}_b^* \ell *} C_L^{\chi_j^0 \widetilde{\ell}_a^* \ell} + (L \longleftrightarrow R)\,,
\end{aligned}$$

$$
\begin{aligned}
C^{abij}_{LRLL} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}_b^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b^*\ell*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} + (L \longleftrightarrow R)\,,\\
C^{abij}_{RLLL} &= C_R^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b^*\ell*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} + (L \longleftrightarrow R)\,,\\
C^{abij}_{LLRL} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b^*\ell} C_R^{\chi_j^0\widetilde{\ell}_b^*\ell*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} + (L \longleftrightarrow R)\,,\\
C^{abij}_{LLLR} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b^*\ell*} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell} + (L \longleftrightarrow R)\,;
\end{aligned}
$$

- $A - Z$ interference:

$$
\widetilde{w}^{(A-Z)}_{\ell\bar{\ell}} = 
$$

$$
= -8\,\mathrm{Re}\left[\left(\frac{C_P^{\ell\ell A} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A}\right)^* \left(\frac{C_A^{\ell\ell Z} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right)\right] \cdot m_\ell \left(m^2_{\widetilde{\ell}_a} - m^2_{\widetilde{\ell}_b}\right)\left(s - m_Z^2\right); \quad (4.69)
$$

- Higgs $(h, H, A)$ - neutralino $(\chi_i^0)$ interference:

$$
\begin{aligned}
\widetilde{w}^{(h,H,A-\chi^0)}_{\ell\bar{\ell}} = 4\,\mathrm{Re} \sum_{i=1}^{4} \sum_{r=h,H} &\left(\frac{C_S^{\ell\ell r} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r}\right)^* \times \\
&\times \left[C^{ba}_{+ii} m_\ell \left\{-2 + \left(m^2_{\widetilde{\ell}_a} + m^2_{\widetilde{\ell}_b} - 2m^2_{\chi_i^0} - 2m_\ell^2\right)\mathcal{F}\right\} + \right.\\
&\left. \quad + C^{ba}_{-ii} m_{\chi_i^0} \left(s - 4m_\ell^2\right)\mathcal{F}\right] + \qquad (4.70)\\
+ 4\,\mathrm{Re}\sum_{i=1}^{4} &\left(\frac{C_P^{\ell\ell A} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a A}}{s - m_A^2 + i\Gamma_A m_A}\right)^* \left[D^{ba}_{+ii} m_\ell \left(m^2_{\widetilde{\ell}_a} - m^2_{\widetilde{\ell}_b}\right) + D^{ba}_{-ii} m_{\chi_i^0} s\right]\mathcal{F}\,,
\end{aligned}
$$

where

$$
C^{ab}_{\pm ij} = C_S^{\chi_i^0\widetilde{\ell}_a^*\ell} \left(C_S^{\chi_j^0\widetilde{\ell}_b^*\ell}\right)^* \pm C_P^{\chi_i^0\widetilde{\ell}_a^*\ell} \left(C_P^{\chi_j^0\widetilde{\ell}_b^*\ell}\right)^*, \qquad (4.71)
$$

$$
D^{ab}_{\pm ij} = C_S^{\chi_i^0\widetilde{\ell}_a^*\ell} \left(C_P^{\chi_j^0\widetilde{\ell}_b^*\ell}\right)^* \pm C_P^{\chi_i^0\widetilde{\ell}_a^*\ell} \left(C_S^{\chi_j^0\widetilde{\ell}_b^*\ell}\right)^*; \qquad (4.72)
$$

- $Z, \gamma$ - neutralino $(\chi_i^0)$ interference:

$$
\begin{aligned}
\widetilde{w}^{(Z,\gamma-\chi^0)}_{\ell\bar{\ell}} = 4\,\mathrm{Re}\sum_{i=1}^{4} &\left(\frac{C_V^{\ell\ell Z} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} - \frac{e e_\ell \delta_{ab}}{s}\right)^* \times \\
&\times \left[C^{ba}_{+ii}\left\{ -s + m^2_{\widetilde{\ell}_a} + m^2_{\widetilde{\ell}_b} - 2m^2_{\chi_i^0} - 2m_\ell^2 + \right.\right.\\
&\left. \quad + 2\left[-s m^2_{\chi_i^0} - \left(m^2_{\widetilde{\ell}_a} - m^2_{\chi_i^0}\right)\left(m^2_{\widetilde{\ell}_b} - m^2_{\chi_i^0}\right) + m_\ell^4\right]\mathcal{F}\right\} + \\
&\left. + C^{ba}_{-ii} \cdot 2 m_\ell m_{\chi_i^0}\left\{-2 + \left(-s + m^2_{\widetilde{\ell}_a} + m^2_{\widetilde{\ell}_b} - 2m^2_{\chi_i^0} + 2m_\ell^2\right)\mathcal{F}\right\}\right] +
\end{aligned}
$$

$$
\begin{aligned}
&+4\,\mathrm{Re}\sum_{i=1}^{4}\left(\frac{C_A^{\ell\ell Z}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right)^*\frac{1}{m_Z^2}\times\\
&\quad\times\Bigg[D_{+ii}^{ba}\bigg\{m_Z^2\left(-s+m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2-2m_{\chi_i^0}^2+2m_\ell^2\right)-\\
&\qquad-2\bigg[m_Z^2\left\{sm_{\chi_i^0}^2+\left(m_{\widetilde{\ell}_a}^2-m_{\chi_i^0}^2\right)\left(m_{\widetilde{\ell}_b}^2-m_{\chi_i^0}^2\right)\right\}+\\
&\qquad+m_\ell^2\bigg\{\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)^2-\\
&\qquad-m_Z^2\left(m_{\widetilde{\ell}_a}^2+m_{\widetilde{\ell}_b}^2+2m_{\chi_i^0}^2-m_\ell^2\right)\bigg\}\bigg]\mathcal{F}\bigg\}-\\
&\qquad-D_{-ii}^{ba}\cdot 2m_\ell m_{\chi_i^0}\left(m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b}^2\right)\left(s-m_Z^2\right)\mathcal{F}\Bigg].
\end{aligned}
\tag{4.73}
$$

### 4.15 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to q\bar{q}$

This process involves the $s$-channel CP-even Higgs boson ($h$ and $H$) and CP-odd Higgs boson ($A$) exchange, and the $s$-channel Z-boson and photon exchange

$$
\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to q\bar{q}}=\widetilde{w}_{q\bar{q}}^{(h,H,A)}+\widetilde{w}_{q\bar{q}}^{(Z,\gamma)}+\widetilde{w}_{q\bar{q}}^{(A-Z)}\ : \tag{4.74}
$$

Each contribution can be found from the respective ones for the $\ell\bar{\ell}$ final state by replacing $\ell$ with $q$.

### 4.16 $\widetilde{\ell}_a\widetilde{\ell}_b^* \to \nu_\ell\bar{\nu}_\ell$

This process involves the $s$-channel Z-boson, and the $t$-channel chargino ($\chi_k^\pm$, $k=1,2$) exchange

$$
\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to\nu_\ell\bar{\nu}_\ell}=\widetilde{w}_{\nu_\ell\bar{\nu}_\ell}^{(Z)}+\widetilde{w}_{\nu_\ell\bar{\nu}_\ell}^{(\chi^\pm)}+\widetilde{w}_{\nu_\ell\bar{\nu}_\ell}^{(Z-\chi^\pm)}\ : \tag{4.75}
$$

Each contribution can be found from the respective ones for the $\ell\bar{\ell}$ final state by replacing $\ell$ with $\nu_\ell$.

### 4.17 $\widetilde{\ell}_a\widetilde{\ell}_b'^* \to \ell\bar{\ell}'$

This process involves the $t$-channel neutralino ($\chi_i^0$, $i=1,2,3,4$) exchange

$$
\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b'^*\to\ell\bar{\ell}'}=\widetilde{w}_{\ell\bar{\ell}'}^{(\chi^0)}\ : \tag{4.76}
$$

- <u>neutralino ($\chi_i^0$) exchange:</u>

$$
\begin{aligned}
\widetilde{w}_{\ell\bar{\ell}'}^{(\chi^0)}=\sum_{i,j=1}^{4}\Bigg[&\widetilde{C}_{LLLL}^{abij}\left[-\mathcal{T}_2^t-\left(s-m_{\widetilde{\ell}_a}^2-m_{\widetilde{\ell}_b'}^2\right)\mathcal{T}_1^t-\left(m_{\widetilde{\ell}_a}^2-m_\ell^2\right)\left(m_{\widetilde{\ell}_b'}^2-m_{\ell'}^2\right)\mathcal{T}_0^t\right]-\\
&-2m_\ell m_{\ell'}\left[\widetilde{C}_{LRLR}^{abij}m_{\chi_i^0}m_{\chi_j^0}\mathcal{T}_0^t+\widetilde{C}_{LLRR}^{abij}\mathcal{T}_1^t\right]+\\
&+\widetilde{C}_{LRRL}^{abij}m_{\chi_i^0}m_{\chi_j^0}\left(s-m_\ell^2-m_{\ell'}^2\right)\mathcal{T}_0^t-
\end{aligned}
$$

$$
\begin{aligned}
&- m_{\chi_i^0}\Big[\widetilde{C}^{abij}_{LRLL} m_{\ell'}\left\{\mathcal{T}_1^t - \left(m^2_{\widetilde{\ell}_a} - m^2_{\ell}\right)\mathcal{T}_0^t\right\} + \\
&\qquad + \widetilde{C}^{abij}_{RLLL} m_{\ell}\left\{\mathcal{T}_1^t - \left(m^2_{\widetilde{\ell}'_b} - m^2_{\ell'}\right)\mathcal{T}_0^t\right\}\Big] - \\
&- m_{\chi_j^0}\Big[\widetilde{C}^{abij}_{LLRL} m_{\ell'}\left\{\mathcal{T}_1^t - \left(m^2_{\widetilde{\ell}_a} - m^2_{\ell}\right)\mathcal{T}_0^t\right\} + \\
&\qquad + \widetilde{C}^{abij}_{LLLR} m_{\ell}\left\{\mathcal{T}_1^t - \left(m^2_{\widetilde{\ell}'_b} - m^2_{\ell'}\right)\mathcal{T}_0^t\right\}\Big]\Bigg],
\end{aligned}
\tag{4.77}
$$

where

$$
\begin{aligned}
\widetilde{C}^{abij}_{LLLL} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longrightarrow R)\,,\\
\widetilde{C}^{abij}_{LRLR} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,,\\
\widetilde{C}^{abij}_{LLRR} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,,\\
\widetilde{C}^{abij}_{LRRL} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,,\\
\widetilde{C}^{abij}_{LRLL} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,,\\
\widetilde{C}^{abij}_{RLLL} &= C_R^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,,\\
\widetilde{C}^{abij}_{LLRL} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,,\\
\widetilde{C}^{abij}_{LLLR} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}'^*_b\ell'} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_j^0\widetilde{\ell}'^*_b\ell'} + (L \longleftrightarrow R)\,.
\end{aligned}
$$

### 4.18 $\widetilde{\ell}_a\widetilde{\ell}'^*_b \to \nu_\ell\bar{\nu}_{\ell'}$

This process involves the $t$-channel chargino ($\chi_k^\pm$, $k = 1, 2$) exchange

$$
\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}'^*_b\to\nu_\ell\bar{\nu}_{\ell'}} = \widetilde{w}^{(\chi^\pm)}_{\nu_\ell\bar{\nu}_{\ell'}} \;: \tag{4.78}
$$

The expression for $\widetilde{w}^{(\chi^\pm)}_{\nu_\ell\bar{\nu}_{\ell'}}$ can be found from $\widetilde{w}^{(\chi^0)}_{\ell\bar{\ell}'}$ by replacing $\ell$ and $\chi^0$ with $\nu_\ell$ and $\chi^\pm$, respectively. The second class of processes involves $\widetilde{\ell}_a\widetilde{\ell}_b \to \ell\ell$ and $\widetilde{\ell}_a\widetilde{\ell}'_b \to \ell\ell'$. For each there is a corresponding process involving particles with the oposite electric charge.

### 4.19 $\widetilde{\ell}_a\widetilde{\ell}_b \to \ell\ell$

This process proceeds only via the $t$- and $u$-channel neutralino ($\chi_i^0$, $i = 1, 2, 3, 4$) exchange

$$
\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}_b\to\ell\ell} = \widetilde{w}^{(\chi^0)}_{\ell\ell} \;: \tag{4.79}
$$

- neutralino ($\chi_i^0$) exchange:

$$
\begin{aligned}
\widetilde{w}^{(\chi^0)}_{\widetilde{\ell}_a\widetilde{\ell}_b\to\ell\ell} = \sum_{i,j=1}^{4}\Bigg[& D^{abij}_{LLLL} m_{\chi_i^0} m_{\chi_j^0}\left(s - 2m_\ell^2\right)\mathcal{T}_0 - \\
&- 2m_\ell^2\Big[D^{abij}_{LLRR} m_{\chi_i^0} m_{\chi_j^0}\mathcal{T}_0 + D^{abij}_{LRRL}\mathcal{T}_1\Big] +
\end{aligned}
$$



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$$
\begin{aligned}
&+ D^{abij}_{LRLR}\Big[- \mathcal{T}_2 - \left(s - m^2_{\tilde{\ell}_a} - m^2_{\tilde{\ell}_b}\right)\mathcal{T}_1 - \\
&\qquad - \left(m^2_{\tilde{\ell}_a} - m^2_{\ell}\right)\left(m^2_{\tilde{\ell}_b} - m^2_{\ell}\right)\mathcal{T}_0\Big] - \\
&- m_\ell m_{\chi^0_i}\Big[D^{abij}_{LLLR}\left\{\mathcal{T}_1 - \left(m^2_{\tilde{\ell}_a} - m^2_{\ell}\right)\mathcal{T}_0\right\} + \\
&\qquad + D^{abij}_{LLRL}\left\{\mathcal{T}_1 - \left(m^2_{\tilde{\ell}_b} - m^2_{\ell}\right)\mathcal{T}_0\right\}\Big] - \\
&- m_\ell m_{\chi^0_j}\Big[D^{abij}_{LRLL}\left\{\mathcal{T}_1 - \left(m^2_{\tilde{\ell}_a} - m^2_{\ell}\right)\mathcal{T}_0\right\} + \\
&\qquad + D^{abij}_{RLLL}\left\{\mathcal{T}_1 - \left(m^2_{\tilde{\ell}_b} - m^2_{\ell}\right)\mathcal{T}_0\right\}\Big]\Big] + \\
&+ \frac{1}{2}\sum_{i,j=1}^{4}\Big[- 2D^{abij}_{LLLL} m_{\chi^0_i} m_{\chi^0_j}\left(s - 2m^2_\ell\right)\mathcal{Y}_0 + \\
&\quad + 4D^{abij}_{LLRR} m_{\chi^0_i} m_{\chi^0_j} m^2_\ell \mathcal{Y}_0 - 2D^{abij}_{LRLR} m^2_\ell\left(m^2_{\tilde{\ell}_a} + m^2_{\tilde{\ell}_b} - 2m^2_\ell\right)\mathcal{Y}_0 + \\
&\quad + 2D^{abij}_{LRRL}\left[-\mathcal{Y}_2 - \left(m^2_{\tilde{\ell}_a} m^2_{\tilde{\ell}_b} - m^4_\ell\right)\mathcal{Y}_0\right] - \\
&\quad - m_\ell m_{\chi^0_i}\Big[D^{abij}_{LLLR}\left\{\mathcal{Y}_1 + \left(s + m^2_{\tilde{\ell}_a} - m^2_{\tilde{\ell}_b} - 4m^2_\ell\right)\mathcal{Y}_0\right\} + \\
&\qquad + D^{abij}_{LLRL}\left\{\mathcal{Y}_1 + \left(s - m^2_{\tilde{\ell}_a} + m^2_{\tilde{\ell}_b} - 4m^2_\ell\right)\mathcal{Y}_0\right\}\Big] + \\
&\quad + m_\ell m_{\chi^0_j}\Big[D^{abij}_{LRLL}\left\{\mathcal{Y}_1 - \left(s + m^2_{\tilde{\ell}_a} - m^2_{\tilde{\ell}_b} - 4m^2_\ell\right)\mathcal{Y}_0\right\} + \\
&\qquad + D^{abij}_{RLLL}\left\{\mathcal{Y}_1 - \left(s - m^2_{\tilde{\ell}_a} + m^2_{\tilde{\ell}_b} - 4m^2_\ell\right)\mathcal{Y}_0\right\}\Big]\Big],
\end{aligned}
\tag{4.80}
$$

where

$$
\begin{aligned}
D^{abij}_{LLLL} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_L^{\chi^0_i \tilde{\ell}^*_b \ell *} C_L^{\chi^0_j \tilde{\ell}^*_b \ell} C_L^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longrightarrow R)\,, \\
D^{abij}_{LLRR} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_L^{\chi^0_i \tilde{\ell}^*_b \ell *} C_R^{\chi^0_j \tilde{\ell}^*_b \ell} C_R^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,, \\
D^{abij}_{LRRL} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_R^{\chi^0_i \tilde{\ell}^*_b \ell *} C_R^{\chi^0_j \tilde{\ell}^*_b \ell} C_L^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,, \\
D^{abij}_{LRLR} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_R^{\chi^0_i \tilde{\ell}^*_b \ell *} C_L^{\chi^0_j \tilde{\ell}^*_b \ell} C_R^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,, \\
D^{abij}_{LLLR} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_L^{\chi^0_i \tilde{\ell}^*_b \ell *} C_L^{\chi^0_j \tilde{\ell}^*_b \ell} C_R^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,, \\
D^{abij}_{LLRL} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_L^{\chi^0_i \tilde{\ell}^*_b \ell *} C_R^{\chi^0_j \tilde{\ell}^*_b \ell} C_L^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,, \\
D^{abij}_{LRLL} &= C_L^{\chi^0_i \tilde{\ell}^*_a \ell *} C_R^{\chi^0_i \tilde{\ell}^*_b \ell *} C_L^{\chi^0_j \tilde{\ell}^*_b \ell} C_L^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,, \\
D^{abij}_{RLLL} &= C_R^{\chi^0_i \tilde{\ell}^*_a \ell *} C_L^{\chi^0_i \tilde{\ell}^*_b \ell *} C_L^{\chi^0_j \tilde{\ell}^*_b \ell} C_L^{\chi^0_j \tilde{\ell}^*_a \ell} + (L \longleftrightarrow R)\,.
\end{aligned}
\tag{4.81}
$$

### 4.20 $\widetilde{\ell}_a\widetilde{\ell}'_b \to \ell\ell'$

This process proceeds only via the $t$-channel neutralino ($\chi_i^0$, $i = 1, 2, 3, 4$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\widetilde{\ell}'_b\to\ell\ell'} = \widetilde{w}_{\ell\ell'}^{(\chi^0)} \;: \tag{4.82}$$

- <u>neutralino ($\chi_i^0$) exchange:</u>

$$\begin{aligned}
\widetilde{w}_{\ell\ell'}^{(\chi^0)} = \sum_{i,j=1}^{4} \Bigg[ & \widetilde{D}_{LLLL}^{abij} m_{\chi_i^0} m_{\chi_j^0} \left(s - m_\ell^2 - m_{\ell'}^2\right) \mathcal{T}_0^t - \\
& - 2 m_\ell m_{\ell'} \left[ \widetilde{D}_{LLRR}^{abij} m_{\chi_i^0} m_{\chi_j^0} \mathcal{T}_0^t + \widetilde{D}_{LRRL}^{abij} \mathcal{T}_1^t \right] + \\
& + \widetilde{D}_{LRLR}^{abij} \left[ -\mathcal{T}_2^t - \left(s - m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}'_b}^2\right) \mathcal{T}_1^t - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}'_b}^2 - m_{\ell'}^2\right) \mathcal{T}_0^t \right] - \\
& - m_{\chi_i^0} \Bigg[ \widetilde{D}_{LLLR}^{abij} m_{\ell'} \left\{ \mathcal{T}_1^t - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right) \mathcal{T}_0^t \right\} + \\
& \qquad + \widetilde{D}_{LLRL}^{abij} m_\ell \left\{ \mathcal{T}_1^t - \left(m_{\widetilde{\ell}'_b}^2 - m_{\ell'}^2\right) \mathcal{T}_0^t \right\} \Bigg] - \\
& - m_{\chi_j^0} \Bigg[ \widetilde{D}_{LRLL}^{abij} m_{\ell'} \left\{ \mathcal{T}_1^t - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right) \mathcal{T}_0^t \right\} + \\
& \qquad + \widetilde{D}_{RLLL}^{abij} m_\ell \left\{ \mathcal{T}_1^t - \left(m_{\widetilde{\ell}'_b}^2 - m_{\ell'}^2\right) \mathcal{T}_0^t \right\} \Bigg] \Bigg] ,
\end{aligned} \tag{4.83}$$

where

$$\begin{aligned}
\widetilde{D}_{LLLL}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longrightarrow R) , \\
\widetilde{D}_{LLRR}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell} C_R^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) , \\
\widetilde{D}_{LRRL}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) , \\
\widetilde{D}_{LRLR}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} C_R^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) , \\
\widetilde{D}_{LLLR}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} C_R^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) , \\
\widetilde{D}_{LLRL}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_R^{\chi_j^0\widetilde{\ell}_a^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) , \\
\widetilde{D}_{LRLL}^{abij} &= C_L^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_R^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) , \\
\widetilde{D}_{RLLL}^{abij} &= C_R^{\chi_i^0\widetilde{\ell}_a^*\ell*} C_L^{\chi_i^0\widetilde{\ell}_b'^*\ell'*} C_L^{\chi_j^0\widetilde{\ell}_a^*\ell} C_L^{\chi_j^0\widetilde{\ell}_b'^*\ell'} + (L \longleftrightarrow R) .
\end{aligned}$$

Finally, there are slepton-neutralino annihilation channels into a lepton and either a gauge or a Higgs boson. For each, there is an analogous one involving the opposite electric charge.

### 4.21 $\widetilde{\ell}_a\chi \to Z\ell$

This process involves the $s$-channel lepton ($\ell$) exchange, the $t$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange, and the $u$-channel neutralino ($\chi_i^0$, $i = 1, 2, 3, 4$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\chi\to Z\ell} = \widetilde{w}_{Z\ell}^{(\ell)} + \widetilde{w}_{Z\ell}^{(\widetilde{\ell})} + \widetilde{w}_{Z\ell}^{(\chi^0)} + \widetilde{w}_{Z\ell}^{(\ell-\widetilde{\ell})} + \widetilde{w}_{Z\ell}^{(\ell-\chi^0)} + \widetilde{w}_{Z\ell}^{(\widetilde{\ell}-\chi^0)} \;: \tag{4.84}$$

- lepton ($\ell$) exchange:

$$
\begin{aligned}
\widetilde{w}_{Z\ell}^{(\ell)} &= \frac{1}{2m_Z^2(s-m_\ell^2)^2} \times \\
&\times \Bigg[ \left\{ C_{+11}^{aa} C_{+}^{\ell\ell Z} + D_{+11}^{aa} D_{+}^{\ell\ell Z} + \frac{m_\ell^2}{s} \left( C_{+11}^{aa} C_{+}^{\ell\ell Z} - D_{+11}^{aa} D_{+}^{\ell\ell Z} \right) \right\} \times \\
&\quad \times \left( s - m_{\widetilde{\ell}_a}^2 + m_\chi^2 \right) \left\{ \left( s - m_\ell^2 \right)^2 + m_Z^2 \left( s + m_\ell^2 \right) - 2m_Z^4 \right\} + \\
&\quad + 4C_{-11}^{aa} C_{+}^{\ell\ell Z} m_\ell m_\chi \left\{ \left( s - m_\ell^2 \right)^2 + m_Z^2 \left( s + m_\ell^2 \right) - 2m_Z^4 \right\} - \\
&\quad - 12 C_{+11}^{aa} C_{-}^{\ell\ell Z} m_\ell^2 m_Z^2 \left( s - m_{\widetilde{\ell}_a}^2 + m_\chi^2 \right) - \\
&\quad - 12 C_{-11}^{aa} C_{-}^{\ell\ell Z} m_\ell m_\chi m_Z^2 \left( s + m_\ell^2 \right) \Bigg], \qquad (4.85)
\end{aligned}
$$

where

$$
C_{\pm}^{\ell\ell Z} = \left( C_V^{\ell\ell Z} \right)^2 \pm \left( C_A^{\ell\ell Z} \right)^2, \qquad (4.86)
$$

$$
D_{+}^{\ell\ell Z} = 2 C_V^{\ell\ell Z} C_A^{\ell\ell Z}; \qquad (4.87)
$$

- slepton ($\widetilde{\ell}_b$) exchange:

$$
\begin{aligned}
\widetilde{w}_{Z\ell}^{(\widetilde{\ell})} &= \frac{1}{m_Z^2} \sum_{b,c=1}^{2} C^{\widetilde{\ell}_b^* \widetilde{\ell}_a Z} C^{\widetilde{\ell}_c^* \widetilde{\ell}_a Z *} \times \\
&\times \Bigg[ C_{+11}^{cb} \Big\{ -\mathcal{T}_3^t + \left( 2m_{\widetilde{\ell}_a}^2 + m_\chi^2 + 2m_Z^2 + m_\ell^2 \right) \mathcal{T}_2^t - \\
&\quad - \Big[ m_{\widetilde{\ell}_a}^4 + 2m_{\widetilde{\ell}_a}^2 \left( m_\chi^2 - m_Z^2 + m_\ell^2 \right) + \\
&\quad + m_Z^2 \left( 2m_\chi^2 + m_Z^2 + 2m_\ell^2 \right) \Big] \mathcal{T}_1^t + \\
&\quad + \left( m_\chi^2 + m_\ell^2 \right) \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right)^2 \mathcal{T}_0^t \Big\} + \\
&\quad + C_{-11}^{cb} \cdot 2m_\ell m_\chi \Big\{ \mathcal{T}_2^t - 2 \left( m_{\widetilde{\ell}_a}^2 + m_Z^2 \right) \mathcal{T}_1^t + \\
&\quad + \left( m_{\widetilde{\ell}_a}^2 - m_Z^2 \right)^2 \mathcal{T}_0^t \Big\} \Bigg]; \qquad (4.88)
\end{aligned}
$$

- neutralino ($\chi_i^0$) exchange:

$$
\begin{aligned}
\widetilde{w}_{Z\ell}^{(\chi^0)} &= \frac{1}{m_Z^2} \sum_{i,j=1}^{4} C_A^{\chi_i^0 \chi Z} C_A^{\chi_j^0 \chi Z *} \times \\
&\times \Bigg[ C_{+ji}^{aa} \Big\{ \mathcal{T}_3^u + \left( s - m_{\widetilde{\ell}_a}^2 - 2m_\chi^2 - m_Z^2 \right) \mathcal{T}_2^u + \\
&\quad + \Big[ -s \left( m_\chi^2 + 2m_Z^2 \right) + m_\chi^4 + m_\chi^2 \left( 2m_{\widetilde{\ell}_a}^2 + 2m_Z^2 - m_\ell^2 \right) + 2m_\ell^2 m_Z^2 \Big] \mathcal{T}_1^u -
\end{aligned}
$$

$$
\begin{aligned}
&- \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right)\left(m_\chi^2 - m_Z^2\right)\left(m_\chi^2 + 2m_Z^2\right)\mathcal{T}_0^u + \\
&+ m_\chi\left(m_{\chi_i^0} + m_{\chi_j^0}\right)\cdot 3m_Z^2\left[\mathcal{T}_1^u - \left(m_{\widetilde{\ell}_a}^2 - m_\ell^2\right)\mathcal{T}_0^u\right] + \\
&+ m_{\chi_i^0}m_{\chi_j^0}\Big[\left(-s + 2m_Z^2 + m_\ell^2\right)\mathcal{T}_1^u + \\
&\qquad + \Big\{s\left(m_\chi^2 + 2m_Z^2\right) - m_Z^2\left(m_{\widetilde{\ell}_a}^2 + m_\chi^2 + 2m_Z^2\right) - \\
&\qquad\qquad - m_\ell^2\left(m_\chi^2 + m_Z^2\right)\Big\}\mathcal{T}_0^u\Big]\Big\} + \\
&+ C_{-ji}^{aa}\cdot m_\ell\Big\{6m_\chi m_Z^2\mathcal{T}_1^u + \left(m_{\chi_i^0} + m_{\chi_j^0}\right)\times \\
&\qquad\times\Big[\mathcal{T}_2^u - \left(2m_\chi^2 - m_Z^2\right)\mathcal{T}_1^u + \\
&\qquad\qquad + \left(m_\chi^2 - m_Z^2\right)\left(m_\chi^2 + 2m_Z^2\right)\mathcal{T}_0^u\Big] + \\
&\qquad + 6m_\chi m_{\chi_i^0}m_{\chi_j^0}m_Z^2\mathcal{T}_0^u\Big\}\Big]; \qquad (4.89)
\end{aligned}
$$

- <u>lepton ($\ell$) - slepton ($\widetilde{\ell}_b$) interference:</u>

$$
\begin{aligned}
\widetilde{w}_{Z\ell}^{(\ell-\widetilde{\ell})} = \frac{1}{m_Z^2}\,\mathrm{Re}\sum_{b=1}^{2}\frac{C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z}}{s - m_\ell^2}\Big[&\left(C_V^{\ell\ell Z*}C_{+11}^{ab} + C_A^{\ell\ell Z*}D_{+11}^{ab}\right)I_1^{Z\ell} - \\
&- \left(C_V^{\ell\ell Z*}C_{+11}^{ab} - C_A^{\ell\ell Z*}D_{+11}^{ab}\right)m_\ell^2 I_2^{Z\ell} + \\
&+ 4C_V^{\ell\ell Z*}C_{-11}^{ab}m_\ell m_\chi I_3^{Z\ell}\Big], \qquad (4.90)
\end{aligned}
$$

where

$$
\begin{aligned}
I_1^{Z\ell} = &-s^2 - s\left(m_{\widetilde{\ell}_a}^2 + m_\chi^2 + m_Z^2 - 2m_{\widetilde{\ell}_b}^2 - m_\ell^2\right) + \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\left(3m_Z^2 - m_\ell^2\right) + \\
&+ 2\Big\{s\left[m_{\widetilde{\ell}_b}^4 - m_{\widetilde{\ell}_b}^2\left(m_{\widetilde{\ell}_a}^2 + m_\chi^2 + m_Z^2\right) + m_\chi^2\left(m_{\widetilde{\ell}_a}^2 - m_Z^2\right)\right] + \\
&\quad + 2m_Z^2\left[m_{\widetilde{\ell}_b}^2\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) + m_\chi^2\left(m_\chi^2 + m_Z^2 - m_{\widetilde{\ell}_a}^2\right)\right] - \\
&\quad - m_\ell^2\left[\left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_a}^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) + m_Z^2\left(m_{\widetilde{\ell}_a}^2 + m_\chi^2\right)\right]\Big\}\mathcal{F}^t, \qquad (4.91) \\
I_2^{Z\ell} = &\; s + 2m_{\widetilde{\ell}_b}^2 + m_\chi^2 - 3m_{\widetilde{\ell}_a}^2 - m_Z^2 - m_\ell^2 - \frac{1}{s}\left(m_Z^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) + \\
&+ 2\Big\{s\left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_a}^2 + m_Z^2\right) + \left(m_{\widetilde{\ell}_b}^2 - m_{\widetilde{\ell}_a}^2\right)^2 - m_{\widetilde{\ell}_b}^2\left(m_Z^2 + m_\ell^2\right) - \\
&\quad - m_{\widetilde{\ell}_a}^2\left(m_Z^2 - m_\ell^2\right) - m_Z^2\left(2m_\chi^2 - m_\ell^2\right)\Big\}\mathcal{F}^t, \qquad (4.92) \\
I_3^{Z\ell} = &-s - m_Z^2 + m_\ell^2 + \Big\{s\left(m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 - m_Z^2\right) - \left(m_{\widetilde{\ell}_b}^2 - m_Z^2\right)\left(m_Z^2 - m_\ell^2\right) - \\
&\qquad - m_{\widetilde{\ell}_a}^2\left(m_Z^2 + m_\ell^2\right) + 2m_Z^2 m_\chi^2\Big\}\mathcal{F}^t; \qquad (4.93)
\end{aligned}
$$

- lepton ($\ell$) - neutralino ($\chi_i^0$) interference:

$$\widetilde{w}_{Z\ell}^{(\ell-\chi^0)} = \frac{1}{m_Z^2}\,\mathrm{Re}\sum_{i=1}^{4}\frac{C_A^{\chi_i^0\chi Z}}{s-m_\ell^2}\Big[\left(C_A^{\ell\ell Z*}C_{+1i}^{aa}+C_V^{\ell\ell Z*}D_{+1i}^{aa}\right)I_4^{Z\ell}+ \\ +\left(C_A^{\ell\ell Z*}C_{-1i}^{aa}+C_V^{\ell\ell Z*}D_{-1i}^{aa}\right)\cdot 2m_\ell I_5^{Z\ell}- \\ -\left(C_A^{\ell\ell Z*}C_{+1i}^{aa}-C_V^{\ell\ell Z*}D_{+1i}^{aa}\right)m_\ell^2 I_6^{Z\ell}- \\ -\left(C_A^{\ell\ell Z*}C_{-1i}^{aa}-C_V^{\ell\ell Z*}D_{-1i}^{aa}\right)\cdot 2m_\ell I_7^{Z\ell}\Big], \tag{4.94}$$

where

$$I_4^{Z\ell} = s^2+s\left(2m_{\chi_i^0}^2+3m_Z^2-m_{\tilde{\ell}_a}^2-m_\chi^2-m_\ell^2\right)-\left(m_{\tilde{\ell}_a}^2-m_\chi^2\right)\left(3m_Z^2-m_\ell^2\right)+ \\ +2\Big\{s^2m_{\chi_i^0}\left(m_\chi+m_{\chi_i^0}\right)+s\Big[-2m_Z^2\left(m_{\tilde{\ell}_a}^2-m_\ell^2\right)+\left(m_{\chi_i^0}^2-m_\chi^2\right)\left(m_{\chi_i^0}^2-m_{\tilde{\ell}_a}^2\right)- \\ -m_\ell^2\left(m_{\chi_i^0}+m_\chi\right)^2+m_Z^2m_{\chi_i^0}\left(m_{\chi_i^0}+m_\chi\right)\Big]- \\ -\left(m_Z^2-m_\ell^2\right)\left(2m_Z^2+m_\ell^2\right)m_\chi m_{\chi_i^0}+2m_Z^2m_{\tilde{\ell}_a}^4- \\ -m_{\tilde{\ell}_a}^2\left[2m_Z^2m_\chi^2+m_\ell^2\left(m_\chi^2+2m_Z^2\right)\right]- \\ -m_{\chi_i^0}^2\left(2m_Z^2-m_\ell^2\right)\left(m_{\tilde{\ell}_a}^2-m_\chi^2\right)+m_\ell^2m_\chi^2\left(m_\chi^2+m_Z^2+m_\ell^2\right)\Big\}\mathcal{F}^u, \tag{4.95}$$

$$I_5^{Z\ell} = m_{\chi_i^0}\left(-s+2m_Z^2+m_\ell^2\right)+3m_Z^2m_\chi+ \\ +\Big\{sm_{\chi_i^0}\left(m_\chi^2+2m_Z^2-m_{\chi_i^0}^2\right)+ \\ +m_{\chi_i^0}\Big[m_Z^2\left(2m_{\chi_i^0}^2-m_\chi^2-m_{\tilde{\ell}_a}^2-2m_Z^2\right)+m_\ell^2\left(m_{\chi_i^0}^2-m_\chi^2-m_Z^2\right)\Big]+ \\ +3m_\chi m_Z^2\left(m_{\chi_i^0}^2-m_{\tilde{\ell}_a}^2+m_\ell^2\right)\Big\}\mathcal{F}^u, \tag{4.96}$$

$$I_6^{Z\ell} = s-2m_{\chi_i^0}^2-m_{\tilde{\ell}_a}^2+3m_\chi^2-3m_Z^2-m_\ell^2-\frac{1}{s}\left(m_Z^2-m_\ell^2\right)\left(m_{\tilde{\ell}_a}^2-m_\chi^2\right)- \\ -2\left\{\left(m_{\chi_i^0}+m_\chi\right)^2-m_Z^2\right\}\left\{\left(m_{\chi_i^0}-m_\chi\right)^2+2m_Z^2\right\}\mathcal{F}^u, \tag{4.97}$$

$$I_7^{Z\ell} = m_\chi\left(s+m_Z^2-m_\ell^2\right)+\Big\{s\left[m_\chi\left(m_{\chi_i^0}^2-m_\chi^2+m_Z^2\right)-3m_Z^2m_{\chi_i^0}\right]+ \\ +m_\chi\Big[m_{\chi_i^0}^2\left(m_Z^2-m_\ell^2\right)+ \\ +m_Z^2\left(m_{\tilde{\ell}_a}^2-2m_\chi^2+2m_Z^2\right)+m_\ell^2\left(m_\chi^2-2m_Z^2\right)\Big]+ \\ +3m_{\chi_i^0}m_Z^2\left(m_{\tilde{\ell}_a}^2-m_\chi^2\right)\Big\}\mathcal{F}^u; \tag{4.98}$$

- slepton ($\widetilde{\ell}_b$) - neutralino ($\chi_i^0$) interference:

$$
\begin{aligned}
&\widetilde{w}_{Z\ell}^{(\widetilde{\ell}-\chi^0)} = \\
&= \frac{1}{m_Z^2}\,\mathrm{Re}\sum_{b=1}^{2}\sum_{i=1}^{4} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z} C_A^{\chi_i^0\chi Z} \times \\
&\qquad \times \Bigg[ D_{+1i}^{ba}\Bigg\{ \mathcal{Y}_3 - \Big[ s + m_{\widetilde{\ell}_a}^2 - m_\chi^2 + m_Z^2 + m_\ell^2 + \\
&\qquad\qquad + \frac{3}{s}\left(m_Z^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\Big]\mathcal{Y}_2 + I_8^{Z\ell}\mathcal{Y}_1 + I_9^{Z\ell}\mathcal{Y}_0 \Bigg\} - \\
&\qquad - D_{-1i}^{ba} m_\ell\left(m_{\chi_i^0} - m_\chi\right)\Bigg\{ -2\mathcal{Y}_2 + \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\mathcal{Y}_1 + \\
&\qquad\qquad + \Big[ s\left(m_{\widetilde{\ell}_a}^2 + m_\chi^2 - 2m_Z^2\right) - m_{\widetilde{\ell}_a}^4 + \\
&\qquad\qquad\quad + m_{\widetilde{\ell}_a}^2\left(3m_Z^2 - m_\ell^2\right) - m_\chi^2\left(m_\chi^2 - 3m_Z^2\right) - \\
&\qquad\qquad\quad - m_\ell^2\left(m_\chi^2 + 2m_Z^2\right)\Big]\mathcal{Y}_0 + \\
&\qquad\qquad + \frac{1}{s}\left(m_Z^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) \times \\
&\qquad\qquad\quad \times \left[\mathcal{Y}_1 - \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\mathcal{Y}_0\right] - \\
&\qquad\qquad - \frac{1}{2s^2}\left(m_Z^2 - m_\ell^2\right)^2\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^2\mathcal{Y}_0\Bigg\}\Bigg],
\end{aligned}
\tag{4.99}
$$

where

$$
\begin{aligned}
I_8^{Z\ell} &= m_\chi m_{\chi_i^0}\left(s + m_Z^2 - m_\ell^2\right) + \frac{1}{2}m_{\widetilde{\ell}_a}^2\left(2m_\chi^2 - 3m_Z^2\right) - \\
&\quad - \frac{1}{2}m_Z^2 m_\chi^2 + \frac{1}{2}m_\ell^2\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2 + 2m_Z^2\right) + \\
&\quad + \frac{1}{2s}\left(m_Z^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2 + m_Z^2 + m_\ell^2\right) + \\
&\quad + \frac{3}{4s^2}\left(m_Z^2 - m_\ell^2\right)^2\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^2, \qquad (4.100)\\
I_9^{Z\ell} &= \frac{1}{2}s\left[m_{\widetilde{\ell}_a}^2\left(3m_Z^2 + 3m_\ell^2 - 2m_\chi^2\right) + m_\chi^2\left(m_Z^2 + m_\ell^2\right) - 6m_\ell^2 m_Z^2\right] - \\
&\quad - m_{\widetilde{\ell}_a}^4\left(m_\chi^2 + m_Z^2 + 2m_\ell^2\right) + m_{\widetilde{\ell}_a}^2\left\{m_\chi^4 + m_Z^2 m_\chi^2 - m_Z^4 + m_\ell^2\left(m_\chi^2 + 4m_Z^2 - 2m_\ell^2\right)\right\} - \\
&\quad - m_Z^2\left\{m_\chi^4 + m_Z^2 m_\chi^2 - m_\ell^2\left(4m_\chi^2 - m_Z^2 - m_\ell^2\right)\right\} - \\
&\quad - \frac{1}{4s}\left(m_Z^2 - m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) \times \\
&\qquad \times \Big\{2m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_a}^2\left(8m_\chi^2 - 3m_Z^2 + 7m_\ell^2\right) + 2m_\chi^4 + 3m_Z^2 m_\chi^2 + 2m_Z^4 + \\
&\qquad\quad + m_\ell^2\left(m_\chi^2 + 8m_Z^2 + 2m_\ell^2\right)\Big\} -
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{s} m_\chi m_{\chi_i^0} \left(s + m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) \left\{ \left(s - m_\ell^2\right)^2 - 2m_Z^2 \left(s + m_\ell^2\right) + m_Z^4 \right\} - \\
&- \frac{1}{4s^2} \left(m_Z^2 - m_\ell^2\right)^2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2 + m_Z^2 + m_\ell^2\right) - \\
&- \frac{1}{4s^3} \left(m_Z^2 - m_\ell^2\right)^3 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^3 .
\end{aligned}
\tag{4.101}
$$

### 4.22 $\widetilde{\ell}_a \chi \to \gamma \ell$

This process proceeds via the $s$-channel lepton ($\ell$) exchange, and the $t$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange

$$
\widetilde{w}_{\widetilde{\ell}_a \chi \to \gamma \ell} = \widetilde{w}_{\gamma \ell}^{(\ell)} + \widetilde{w}_{\gamma \ell}^{(\widetilde{\ell})} + \widetilde{w}_{\gamma \ell}^{(\ell - \widetilde{\ell})} \ : \tag{4.102}
$$

- <u>lepton ($\ell$) exchange:</u>

$$
\begin{aligned}
\widetilde{w}_{\gamma \ell}^{(\ell)} = \frac{e^2}{\left(s - m_\ell^2\right)^2} \Bigg[ & C_{+11}^{aa} \left(s^2 - 6m_\ell^2 s + m_\ell^4\right) \left\{ 1 - \frac{1}{s} \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right) \right\} - \\
& - C_{-11}^{aa} \cdot 4 m_\ell m_\chi \left(s + m_\ell^2\right) \Bigg] ;
\end{aligned}
\tag{4.103}
$$

- <u>slepton ($\widetilde{\ell}_a$) exchange:</u>

$$
\begin{aligned}
\widetilde{w}_{\gamma \ell}^{(\widetilde{\ell})} = 2e^2 \Bigg[ & C_{+11}^{aa} \left\{ \mathcal{T}_2^t + \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2 - m_\ell^2\right) \mathcal{T}_1^t - \left(m_\chi^2 + m_\ell^2\right) m_{\widetilde{\ell}_a}^2 \mathcal{T}_0^t \right\} - \\
& - C_{-11}^{aa} \cdot 2 m_\ell m_\chi \left\{ \mathcal{T}_1^t + m_{\widetilde{\ell}_a}^2 \mathcal{T}_0^t \right\} \Bigg] ;
\end{aligned}
\tag{4.104}
$$

- <u>lepton ($\ell$) - slepton ($\widetilde{\ell}_a$) interference:</u>

$$
\begin{aligned}
\widetilde{w}_{\gamma \ell}^{(\ell - \widetilde{\ell})} = & - \frac{2e^2}{s - m_\ell^2} \times \\
& \times \mathrm{Re} \Bigg[ C_{+11}^{aa} \Bigg\{ s + 2m_\chi^2 - 2m_{\widetilde{\ell}_a}^2 - 2m_\ell^2 + \\
& \qquad + \Big[ s \left(m_{\widetilde{\ell}_a}^2 + m_\chi^2 + m_\ell^2\right) - 2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^2 - \\
& \qquad - m_\ell^2 \left(m_{\widetilde{\ell}_a}^2 + m_\chi^2 - m_\ell^2\right) \Big] \mathcal{F}^t \Bigg\} + \\
& \quad + C_{-11}^{aa} \cdot 2 m_\ell m_\chi \Bigg\{ 1 + \left(s + 2m_{\widetilde{\ell}_a}^2 - 2m_\chi^2 + m_\ell^2\right) \mathcal{F}^t \Bigg\} \Bigg] .
\end{aligned}
\tag{4.105}
$$

### 4.23 $\widetilde{\ell}_a \chi \to W^- \nu_\ell$

This process proceeds via the $s$-channel lepton ($\ell$) exchange, the $t$-channel sneutrino ($\widetilde{\nu}_\ell$) exchange, and the $u$-channel chargino ($\chi_k^\pm$, $k = 1, 2$) exchange

$$
\widetilde{w}_{\widetilde{\ell}_a \chi \to W \nu_\ell} = \widetilde{w}_{W \nu_\ell}^{(\ell)} + \widetilde{w}_{W \nu_\ell}^{(\widetilde{\nu})} + \widetilde{w}_{W \nu_\ell}^{(\chi^\pm)} + \widetilde{w}_{W \nu_\ell}^{(\ell - \widetilde{\nu})} + \widetilde{w}_{W \nu_\ell}^{(\ell - \chi^\pm)} + \widetilde{w}_{W \nu_\ell}^{(\widetilde{\nu} - \chi^\pm)} \ : \tag{4.106}
$$

- lepton ($\ell$) exchange:

$$\widetilde{w}^{(\ell)}_{W\nu_\ell} = \frac{(C^{\ell\nu_\ell W^+})^2}{m_W^2\left(s-m_\ell^2\right)^2}\left(s-m_W^2\right)\left(s+2m_W^2\right)\times \tag{4.107}$$
$$\times\left[\left\{\left|C_L^{\chi\widetilde{\ell}_a\ell}\right|^2+\frac{m_\ell^2}{s}\left|C_R^{\chi\widetilde{\ell}_a\ell}\right|^2\right\}\left(s-m^2_{\widetilde{\ell}_a}+m_\chi^2\right)+C^{aa}_{-11}\cdot 4m_\ell m_\chi\right];$$

- sneutrino ($\widetilde{\nu}_\ell$) exchange:

$$\widetilde{w}^{(\widetilde{\nu})}_{W\nu_\ell} = \frac{2}{m_W^2}\left|C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a W^+}C_L^{\chi\widetilde{\nu}_\ell\nu_\ell}\right|^2\times$$
$$\times\left[-\mathcal{T}_3^t+\left(m_\chi^2+2m^2_{\widetilde{\ell}_a}+2m_W^2\right)\mathcal{T}_2^t- \right. \tag{4.108}$$
$$\left.-\left\{\left(m^2_{\widetilde{\ell}_a}-m_W^2\right)^2+2m_\chi^2\left(m^2_{\widetilde{\ell}_a}+m_W^2\right)\right\}\mathcal{T}_1^t+m_\chi^2\left(m^2_{\widetilde{\ell}_a}-m_W^2\right)^2\mathcal{T}_0^t\right];$$

- chargino ($\chi_k^\pm$) exchange:

$$\widetilde{w}^{(\chi^\pm)}_{W\nu_\ell} = \frac{2}{m_W^2}\sum_{k,l=1}^{2}C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell *}C_L^{\chi_l^+\widetilde{\ell}_a\nu_\ell}\times$$
$$\times\left[C_L^{\chi_k^+\chi W^- *}C_L^{\chi_l^+\chi W^-}\times\right.$$
$$\times\left\{\mathcal{T}_3^u+\left(s-m^2_{\widetilde{\ell}_a}-2m_\chi^2-m_W^2\right)\mathcal{T}_2^u+\right.$$
$$+\left[-s\left(m_\chi^2+2m_W^2\right)+m_\chi^2\left(2m^2_{\widetilde{\ell}_a}+m_\chi^2+2m_W^2\right)\right]\mathcal{T}_1^u-$$
$$\left.-\,m^2_{\widetilde{\ell}_a}\left(m_\chi^2-m_W^2\right)\left(m_\chi^2+2m_W^2\right)\mathcal{T}_0^u\right\}+$$
$$+\,C_R^{\chi_k^+\chi W^- *}C_R^{\chi_l^+\chi W^-}\times$$
$$\times\, m_{\chi_k^\pm}m_{\chi_l^\pm}\left\{-(s-2m_W^2)\mathcal{T}_1^u+\right.$$
$$+\left[s\left(m_\chi^2+2m_W^2\right)-\right.$$
$$\left.\left.-\,m_W^2\left(m^2_{\widetilde{\ell}_a}+m_\chi^2+2m_W^2\right)\right]\mathcal{T}_0^u\right\}+$$
$$+\,m_\chi\left(m_{\chi_k^\pm}C_R^{\chi_k^+\chi W^- *}C_L^{\chi_l^+\chi W^-}+\right. \tag{4.109}$$
$$\left.\left.+\,m_{\chi_l^\pm}C_L^{\chi_k^+\chi W^- *}C_R^{\chi_l^+\chi W^-}\right)\cdot 3m_W^2\left\{-\mathcal{T}_1^u+m^2_{\widetilde{\ell}_a}\mathcal{T}_0^u\right\}\right];$$

- lepton ($\ell$) - sneutrino ($\widetilde{\nu}_\ell$) interference:

$$\begin{aligned}
\widetilde{w}^{(\ell-\widetilde{\nu})}_{W\nu_\ell} = & -\frac{2}{m_W^2}\,\mathrm{Re}\,\frac{C^{\ell\nu_\ell W^+ *}C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a W^+}C_L^{\chi\widetilde{\nu}_\ell\nu_\ell}}{s-m_\ell^2}\times \\
& \times\Bigg[C_L^{\chi\widetilde{\ell}_a\ell}\Bigg\{s^2+s\left(m^2_{\widetilde{\ell}_a}+m^2_\chi+m^2_W-2m^2_{\widetilde{\nu}_\ell}\right)-3m^2_W\left(m^2_{\widetilde{\ell}_a}-m^2_\chi\right)- \\
& \qquad -\,2\Bigg[s\left\{m^4_{\widetilde{\nu}_\ell}-m^2_{\widetilde{\nu}_\ell}\left(m^2_{\widetilde{\ell}_a}+m^2_\chi+m^2_W\right)+m^2_\chi\left(m^2_{\widetilde{\ell}_a}-m^2_W\right)\right\}+ \\
& \qquad\quad +\,2m^2_W\left\{m^2_{\widetilde{\nu}_\ell}\left(m^2_{\widetilde{\ell}_a}-m^2_\chi\right)+m^2_\chi\left(m^2_\chi+m^2_W-m^2_{\widetilde{\ell}_a}\right)\right\}\Bigg]\mathcal{F}^t\Bigg\}+ \\
& \quad +\,C_R^{\chi\widetilde{\ell}_a\ell}\cdot 2m_\ell m_\chi\Bigg\{s+m^2_W+ \\
& \qquad +\,\left[s\left(m^2_{\widetilde{\nu}_\ell}+m^2_W-m^2_{\widetilde{\ell}_a}\right)+\right. \\
& \qquad\quad \left.+\,m^2_W\left(m^2_{\widetilde{\ell}_a}+m^2_{\widetilde{\nu}_\ell}-2m^2_\chi-m^2_W\right)\right]\mathcal{F}^t\Bigg\}\Bigg]; \qquad (4.110)
\end{aligned}$$

- lepton ($\ell$) - chargino ($\chi_k^\pm$) interference:

$$\begin{aligned}
\widetilde{w}^{(\ell-\chi^\pm)}_{W\nu_\ell} = & \frac{2}{m_W^2}\,\mathrm{Re}\sum_{k=1}^{2}\frac{C^{\ell\nu_\ell W^+ *}C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell}}{s-m_\ell^2}\times \\
& \times\Bigg[C_L^{\chi\widetilde{\ell}_a\ell}C_L^{\chi_k^+\chi W^- *}\Bigg\{s^2+s\left(2m^2_{\chi_k^\pm}+3m^2_W-m^2_{\widetilde{\ell}_a}-m^2_\chi\right)- \\
& \qquad -\,3m^2_W(m^2_{\widetilde{\ell}_a}-m^2_\chi)+ \\
& \qquad +\,2\Bigg[s^2m^2_{\chi_k^\pm}+s\Big\{\left(m^2_{\chi_k^\pm}-m^2_\chi\right)\left(m^2_{\chi_k^\pm}-m^2_{\widetilde{\ell}_a}\right)+ \\
& \qquad\qquad +\,m^2_W\left(m^2_{\chi_k^\pm}-2m^2_{\widetilde{\ell}_a}\right)\Big\}+ \\
& \qquad +\,2m^2_W\left(m^2_{\widetilde{\ell}_a}-m^2_\chi\right)\left(m^2_{\widetilde{\ell}_a}-m^2_{\chi_k^\pm}\right)\Bigg]\mathcal{F}^u\Bigg\}+ \\
& +\,C_R^{\chi\widetilde{\ell}_a\ell}C_R^{\chi_k^+\chi W^- *}\cdot \\
& \cdot\,2m_\ell m_{\chi_k^\pm}\Bigg\{s-2m^2_W+\Bigg[s\left(m^2_{\chi_k^\pm}-m^2_\chi-2m^2_W\right)+ \\
& \qquad +\,m^2_W\Big(m^2_{\widetilde{\ell}_a}+m^2_\chi+ \\
& \qquad\qquad +\,2m^2_W-2m^2_{\chi_k^\pm}\Big)\Bigg]\mathcal{F}^u\Bigg\}+ \\
& +\,C_R^{\chi\widetilde{\ell}_a\ell}C_L^{\chi_k^+\chi W^- *}\cdot 6m_\ell m_\chi m^2_W\left\{1+\left(m^2_{\chi_k^\pm}-m^2_{\widetilde{\ell}_a}\right)\mathcal{F}^u\right\}- \\
& -\,C_L^{\chi\widetilde{\ell}_a\ell}C_R^{\chi_k^+\chi W^- *}\cdot 2m_\chi m_{\chi_k^\pm}\left(s-m^2_W\right)\left(s+2m^2_W\right)\mathcal{F}^u\Bigg]; \qquad (4.111)
\end{aligned}$$

• sneutrino ($\widetilde{\nu}_\ell$) - chargino ($\chi_k^\pm$) interference:

$$\begin{aligned}
\widetilde{w}_{W\nu_\ell}^{(\widetilde{\nu}-\chi^\pm)} = \frac{2}{m_W^2}\,\mathrm{Re}\sum_{k=1}^{2} C^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a W^{+*}} C_L^{\chi\widetilde{\nu}_\ell\nu_\ell} C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell} \times \\
\times \Bigg[ C_L^{\chi_k^+\chi W^{-*}} \Bigg\{ \mathcal{Y}_3 - \Big[s+m_{\widetilde{\ell}_a}^2 - m_\chi^2 + m_W^2 + \frac{3}{s} m_W^2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\Big]\mathcal{Y}_2 + \\
+ \frac{1}{2}\Big[ m_{\widetilde{\ell}_a}^2 \left(2m_\chi^2 - 3m_W^2\right) - m_W^2 m_\chi^2 + \\
+ \frac{3}{2s^2} m_W^4 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^2 + \\
+ \frac{1}{s} m_W^2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2 + m_W^2\right)\Big]\mathcal{Y}_1 + \\
+ I^{W\nu}\mathcal{Y}_0 \Bigg\} + \\
+ C_R^{\chi_k^+\chi W^{-*}} m_\chi m_{\chi_k^\pm} \Bigg\{ - \left(s + m_W^2\right)\mathcal{Y}_1 + \\
+ \left(s - m_W^2\right)^2 \Big[1 + \frac{1}{s}\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\Big]\mathcal{Y}_0 \Bigg\}\Bigg],
\end{aligned} \tag{4.112}$$

where

$$\begin{aligned}
I^{W\nu} = \frac{1}{2}s\left[m_{\widetilde{\ell}_a}^2 \left(3m_W^2 - 2m_\chi^2\right) + m_W^2 m_\chi^2\right] - \\
- m_{\widetilde{\ell}_a}^4 \left(m_\chi^2 + m_W^2\right) + m_{\widetilde{\ell}_a}^2 \left(m_\chi^4 + m_W^2 m_\chi^2 - m_W^4\right) - m_W^2 m_\chi^2 \left(m_\chi^2 + m_W^2\right) - \\
- \frac{1}{4s} m_W^2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\left\{2m_{\widetilde{\ell}_a}^4 + m_{\widetilde{\ell}_a}^2\left(8m_\chi^2 - 3m_W^2\right) + 2m_\chi^4 + 3m_W^2 m_\chi^2 + 2m_W^4\right\} - \\
- \frac{1}{4s^2} m_W^4 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^2 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2 + m_W^2\right) - \frac{1}{4s^3} m_W^6 \left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)^3.
\end{aligned} \tag{4.113}$$

### 4.24 $\widetilde{\ell}_a\chi \to h\ell, H\ell$

The process $\widetilde{\ell}_a\chi \to h\ell$ involves the $s$-channel lepton ($\ell$) exchange, the $t$-channel slepton ($\widetilde{\ell}_a$, $a = 1, 2$) exchange, and the $u$-channel neutralino ($\chi_i^0$, $i = 1, 2, 3, 4$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\chi\to h\ell} = \widetilde{w}_{h\ell}^{(\ell)} + \widetilde{w}_{h\ell}^{(\widetilde{\ell})} + \widetilde{w}_{h\ell}^{(\chi^0)} + \widetilde{w}_{h\ell}^{(\ell-\widetilde{\ell})} + \widetilde{w}_{h\ell}^{(\ell-\chi^0)} + \widetilde{w}_{h\ell}^{(\widetilde{\ell}-\chi^0)} : \tag{4.114}$$

• lepton ($\ell$) exchange:

$$\begin{aligned}
\widetilde{w}_{h\ell}^{(\ell)} = \frac{1}{2}\left|\frac{C_S^{\ell\ell h}}{s - m_\ell^2}\right|^2 \Big[ 4C_{-11}^{aa} m_\ell m_\chi \left(2s + 2m_\ell^2 - m_h^2\right) + \\
+ C_{+11}^{aa}\Big\{ s^2 - s\left(m_h^2 - 6m_\ell^2\right) - m_\ell^2\left(m_h^2 - m_\ell^2\right)\Big\} \times \\
\times \left\{1 - \frac{1}{s}\left(m_{\widetilde{\ell}_a}^2 - m_\chi^2\right)\right\}\Big];
\end{aligned} \tag{4.115}$$

- slepton ($\widetilde{\ell}_b$) exchange:

$$\widetilde{w}_{h\ell}^{(\widetilde{\ell})} = \sum_{b,c=1}^{2} C_S^{\widetilde{\ell}_b^* \widetilde{\ell}_a h} C_S^{\widetilde{\ell}_c^* \widetilde{\ell}_a h*} \Big[ C_{+11}^{bc} \left\{ -\mathcal{T}_1^t + \left( m_\chi^2 + m_\ell^2 \right) \mathcal{T}_0^t \right\} + 2 C_{-11}^{bc} m_\ell m_\chi \mathcal{T}_0^t \Big] ; \qquad (4.116)$$

- neutralino ($\chi_i^0$) exchange:

$$\begin{aligned} \widetilde{w}_{h\ell}^{(\chi^0)} = \sum_{i,j=1}^{4} C_S^{\chi_i^0 \chi h} C_S^{\chi_j^0 \chi h*} \Big[ C_{+ji}^{aa} \Big\{ & - \left( s - m_\chi^2 - m_\ell^2 \right) \mathcal{T}_1^u - \left( m_{\widetilde{\ell}_a}^2 - m_\ell^2 \right) \left( m_\chi^2 - m_h^2 \right) \mathcal{T}_0^u + \\ & + m_\chi \left( m_{\chi_i^0} + m_{\chi_j^0} \right) \left[ \mathcal{T}_1^u - \left( m_{\widetilde{\ell}_a}^2 - m_\ell^2 \right) \mathcal{T}_0^u \right] + \\ & + m_{\chi_i^0} m_{\chi_j^0} \left[ \mathcal{T}_1^u + \left( s - m_{\widetilde{\ell}_a}^2 - m_h^2 \right) \mathcal{T}_0^u \right] \Big\} + \\ & + C_{-ji}^{aa} m_\ell \Big\{ 2 m_\chi \mathcal{T}_1^u + \left( m_{\chi_i^0} + m_{\chi_j^0} \right) \left[ \mathcal{T}_1^u + \left( m_\chi^2 - m_h^2 \right) \mathcal{T}_0^u \right] + \\ & + m_{\chi_i^0} m_{\chi_j^0} \cdot 2 m_\chi \mathcal{T}_0^u \Big\} \Big] ; \end{aligned} \qquad (4.117)$$

- lepton ($\ell$) - slepton ($\widetilde{\ell}_b$) interference:

$$\begin{aligned} \widetilde{w}_{h\ell}^{(\ell-\widetilde{\ell})} = 2 \,\mathrm{Re} \sum_{b=1}^{2} \frac{C_S^{\ell\ell h*} C_S^{\widetilde{\ell}_b^* \widetilde{\ell}_a h}}{s - m_\ell^2} \Big[ & C_{-11}^{ab} m_\chi (s - m_h^2 + 3 m_\ell^2) \mathcal{F}^t + \qquad (4.118) \\ & + C_{+11}^{ab} m_\ell \Big\{ -1 + \left( s + 2 m_\chi^2 + m_\ell^2 - m_{\widetilde{\ell}_a}^2 - m_{\widetilde{\ell}_b}^2 \right) \mathcal{F}^t \Big\} \Big] ; \end{aligned}$$

- lepton ($\ell$) - neutralino ($\chi_i^0$) interference:

$$\begin{aligned} \widetilde{w}_{h\ell}^{(\ell-\chi^0)} = 2 \,\mathrm{Re} \sum_{i=1}^{4} & \frac{C_S^{\ell\ell h*} C_S^{\chi_i^0 \chi h}}{s - m_\ell^2} \times \\ \times \Big[ & C_{+1i}^{aa} m_\ell \Big\{ m_{\chi_i^0} + m_\chi + \Big[ 2 m_{\chi_i^0} s + \left( m_{\chi_i^0} + m_\chi \right) \left( m_{\chi_i^0}^2 + m_\chi^2 - 2 m_{\widetilde{\ell}_a}^2 - m_h^2 \right) + \\ & + 2 m_\chi m_\ell^2 \Big] \mathcal{F}^u \Big\} + \\ & + C_{-1i}^{aa} \Big\{ s + m_\ell^2 + \Big[ s m_{\chi_i^0} \left( m_{\chi_i^0} + m_\chi \right) - m_\chi m_{\chi_i^0} \left( m_h^2 - 3 m_\ell^2 \right) - \\ & - m_{\widetilde{\ell}_a}^2 m_h^2 + m_\ell^2 \left( m_{\chi_i^0}^2 + 2 m_\chi^2 - m_h^2 \right) \Big] \mathcal{F}^u \Big\} \Big] ; \end{aligned} \qquad (4.119)$$

- slepton ($\widetilde{\ell}_b$) - neutralino ($\chi_i^0$) interference:

$$\begin{aligned} \widetilde{w}_{h\ell}^{(\widetilde{\ell}-\chi^0)} = \mathrm{Re} \sum_{b=1}^{2} \sum_{i=1}^{4} & C_S^{\widetilde{\ell}_b^* \widetilde{\ell}_a h*} C_S^{\chi_i^0 \chi h} \times \\ \times \Big[ & C_{+1i}^{ba} \Big\{ - \left( m_{\chi_i^0} + m_\chi \right) \mathcal{Y}_1 + \end{aligned}$$

$$+\left[-m_{\chi_i^0}\left(s-m_h^2+m_\ell^2\right)\left\{1-\frac{1}{s}\left(m_{\widetilde{\ell}_a}^2-m_\chi^2\right)\right\}+\right.$$

$$+m_\chi\left\{s+m_{\widetilde{\ell}_a}^2-m_h^2-m_\chi^2-3m_\ell^2-\right.$$

$$\left.\left.-\frac{1}{s}\left(m_h^2-m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2-m_\chi^2\right)\right\}\right]\mathcal{Y}_0\Big\}+$$

$$+C_{-1i}^{ba}m_\ell\left\{\mathcal{Y}_1+\left[s-m_{\widetilde{\ell}_a}^2+m_h^2-3m_\chi^2-m_\ell^2-4m_\chi m_{\chi_i^0}-\right.\right.$$

$$\left.\left.\left.-\frac{1}{s}\left(m_h^2-m_\ell^2\right)\left(m_{\widetilde{\ell}_a}^2-m_\chi^2\right)\right]\mathcal{Y}_0\right\}\right]. \tag{4.120}$$

The contribution from $\widetilde{\ell}_a\chi\to H\ell$ can be obtained by a simple substitution $h\to H$.

### 4.25 $\widetilde{\ell}_a\chi\to A\ell$

This process involves the $s$-channel lepton ($\ell$) exchange, the $t$-channel slepton ($\widetilde{\ell}_a$, $a=1,2$) exchange, and the $u$-channel neutralino ($\chi_i^0$, $i=1,2,3,4$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\chi\to A\ell}=\widetilde{w}_{A\ell}^{(\ell)}+\widetilde{w}_{A\ell}^{(\widetilde{\ell})}+\widetilde{w}_{A\ell}^{(\chi^0)}+\widetilde{w}_{A\ell}^{(\ell-\widetilde{\ell})}+\widetilde{w}_{A\ell}^{(\ell-\chi^0)}+\widetilde{w}_{A\ell}^{(\widetilde{\ell}-\chi^0)}\ : \tag{4.121}$$

- lepton ($\ell$) exchange:

$$\widetilde{w}_{A\ell}^{(\ell)}=\frac{1}{2}\left|\frac{C_P^{\ell\ell A}}{s-m_\ell^2}\right|^2\left[-4C_{-11}^{aa}m_\ell m_\chi m_A^2+C_{+11}^{aa}\left\{\left(s-m_\ell^2\right)^2-m_A^2\left(s+m_\ell^2\right)\right\}\times\right.$$

$$\left.\times\left\{1-\frac{1}{s}\left(m_{\widetilde{\ell}_a}^2-m_\chi^2\right)\right\}\right]; \tag{4.122}$$

- slepton ($\widetilde{\ell}_b$) exchange:

$$\widetilde{w}_{A\ell}^{(\widetilde{\ell})}=\sum_{b,c=1}^{2}C_P^{\widetilde{\ell}_b^*\widetilde{\ell}_aA}C_P^{\widetilde{\ell}_c^*\widetilde{\ell}_aA*}\left[C_{+11}^{bc}\left\{-\mathcal{T}_1^t+\left(m_\chi^2+m_\ell^2\right)\mathcal{T}_0^t\right\}+2C_{-11}^{bc}m_\ell m_\chi\mathcal{T}_0^t\right]; \tag{4.123}$$

- neutralino ($\chi_i^0$) exchange:

$$\widetilde{w}_{A\ell}^{(\chi^0)}=\sum_{i,j=1}^{4}C_P^{\chi_i^0\chi A}C_P^{\chi_j^0\chi A*}\left[C_{+ji}^{aa}\left\{-\left(s-m_\chi^2-m_\ell^2\right)\mathcal{T}_1^u-\left(m_{\widetilde{\ell}_a}^2-m_\ell^2\right)\left(m_\chi^2-m_A^2\right)\mathcal{T}_0^u-\right.\right.$$

$$-m_\chi\left(m_{\chi_i^0}+m_{\chi_j^0}\right)\left[\mathcal{T}_1^u-\left(m_{\widetilde{\ell}_a}^2-m_\ell^2\right)\mathcal{T}_0^u\right]+$$

$$\left.+m_{\chi_i^0}m_{\chi_j^0}\left[\mathcal{T}_1^u+\left(s-m_{\widetilde{\ell}_a}^2-m_A^2\right)\mathcal{T}_0^u\right]\right\}-$$

$$-C_{-ji}^{aa}m_\ell\left\{2m_\chi\mathcal{T}_1^u-\left(m_{\chi_i^0}+m_{\chi_j^0}\right)\left[\mathcal{T}_1^u+\left(m_\chi^2-m_A^2\right)\mathcal{T}_0^u\right]+\right.$$

$$\left.\left.+m_{\chi_i^0}m_{\chi_j^0}\cdot 2m_\chi\mathcal{T}_0^u\right\}\right]; \tag{4.124}$$

- lepton ($\ell$) - slepton ($\widetilde{\ell}_b$) interference:

$$\widetilde{w}_{A\ell}^{(\ell-\widetilde{\ell})} = 2\,\mathrm{Re}\sum_{b=1}^{2}\frac{C_P^{\ell\ell A*}C_P^{\widetilde{\ell}_b^*\widetilde{\ell}_a A}}{s-m_\ell^2}\Big[D_{-11}^{ab}m_\chi\left(s-m_A^2-m_\ell^2\right)\mathcal{F}^t + \\ + D_{+11}^{ab}m_\ell\Big\{-1-\Big(s+m_{\widetilde{\ell}_b}^2-m_{\widetilde{\ell}_a}^2-m_\ell^2\Big)\mathcal{F}^t\Big\}\Big]; \quad (4.125)$$

- lepton ($\ell$) - neutralino ($\chi_i^0$) interference:

$$\widetilde{w}_{A\ell}^{(\ell-\chi^0)} = 2\,\mathrm{Re}\sum_{i=1}^{4}\frac{C_P^{\ell\ell A*}C_P^{\chi_i^0\chi A}}{s-m_\ell^2}\Big[C_{+1i}^{aa}m_\ell\Big\{m_{\chi_i^0}-m_\chi+\Big[\Big(m_{\chi_i^0}-m_\chi\Big)\Big(m_{\chi_i^0}^2-m_\chi^2\Big)- \\ -\Big(m_{\chi_i^0}+m_\chi\Big)m_A^2\Big]\mathcal{F}^u\Big\}+ \\ + C_{-1i}^{aa}\Big\{-s+m_\ell^2+\Big[-sm_{\chi_i^0}\Big(m_{\chi_i^0}-m_\chi\Big)- \\ - m_\chi m_{\chi_i^0}\left(m_A^2+m_\ell^2\right)+ \quad (4.126) \\ + m_{\widetilde{\ell}_a}^2m_A^2+m_\ell^2\Big(m_{\chi_i^0}^2-m_A^2\Big)\Big]\mathcal{F}^u\Big\}\Big];$$

- slepton ($\widetilde{\ell}_b$) - neutralino ($\chi_i^0$) interference:

$$\widetilde{w}_{A\ell}^{(\widetilde{\ell}-\chi^0)} = \mathrm{Re}\sum_{b=1}^{2}\sum_{i=1}^{4}C_P^{\widetilde{\ell}_b^*\widetilde{\ell}_a A*}C_P^{\chi_i^0\chi A}\Big[D_{+1i}^{ba}\Big\{\Big(m_{\chi_i^0}-m_\chi\Big)\mathcal{Y}_1+ \\ +\Big[-m_{\chi_i^0}\left(s-m_A^2+m_\ell^2\right)\times \\ \times\Big\{1-\frac{1}{s}\Big(m_{\widetilde{\ell}_a}^2-m_\chi^2\Big)\Big\}- \quad (4.127) \\ - m_\chi\Big\{s+m_{\widetilde{\ell}_a}^2-m_A^2-m_\chi^2-3m_\ell^2- \\ -\frac{1}{s}\left(m_A^2-m_\ell^2\right)\Big(m_{\widetilde{\ell}_a}^2-m_\chi^2\Big)\Big\}\Big]\mathcal{Y}_0\Big\}- \\ - D_{-1i}^{ba}m_\ell\Big\{\mathcal{Y}_1+\Big[s-m_{\widetilde{\ell}_a}^2+m_A^2-3m_\chi^2-m_\ell^2+4m_\chi m_{\chi_i^0}- \\ -\frac{1}{s}\left(m_A^2-m_\ell^2\right)\Big(m_{\widetilde{\ell}_a}^2-m_\chi^2\Big)\Big]\mathcal{Y}_0\Big\}\Big].$$

### 4.26 $\widetilde{\ell}_a\chi \to H^-\nu_\ell$

This process proceeds via the $s$-channel lepton ($\ell$) exchange, the $t$-channel sneutrino ($\widetilde{\nu}_\ell$) exchange, and the $u$-channel chargino ($\chi_k^\pm$, $k=1,2$) exchange

$$\widetilde{w}_{\widetilde{\ell}_a\chi\to H^-\nu_\ell} = \widetilde{w}_{H^-\nu_\ell}^{(\ell)}+\widetilde{w}_{H^-\nu_\ell}^{(\widetilde{\nu})}+\widetilde{w}_{H^-\nu_\ell}^{(\chi^\pm)}+\widetilde{w}_{H^-\nu_\ell}^{(\ell-\widetilde{\nu})}+\widetilde{w}_{H^-\nu_\ell}^{(\ell-\chi^\pm)}+\widetilde{w}_{H^-\nu_\ell}^{(\widetilde{\nu}-\chi^\pm)}: \quad (4.128)$$



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- lepton ($\ell$) exchange:

$$\widetilde{w}^{(\ell)}_{H^-\nu_\ell} = \left|\frac{C_S^{\nu_\ell \ell H^+}}{s-m_\ell^2}\right|^2 (s-m^2_{H^\pm}) \times \\ \times \left[4C^{aa}_{-11} m_\ell m_\chi + \left(s\left|C_R^{\chi\widetilde{\ell}_a\ell}\right|^2 + m_\ell^2\left|C_L^{\chi\widetilde{\ell}_a\ell}\right|^2\right)\left\{1-\frac{1}{s}\left(m^2_{\widetilde{\ell}_a}-m^2_\chi\right)\right\}\right]; \quad (4.129)$$

- sneutrino ($\widetilde{\nu}_\ell$) exchange:

$$\widetilde{w}^{(\widetilde{\nu})}_{H^-\nu_\ell} = 2\left|C_S^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a H^+} C_L^{\chi\widetilde{\nu}_\ell\nu_\ell}\right|^2\left[-\mathcal{T}_1^t + m_\chi^2\mathcal{T}_0^t\right]; \quad (4.130)$$

- chargino ($\chi_k^\pm$) exchange:

$$\widetilde{w}^{(\chi^\pm)}_{H^-\nu_\ell} = 2\sum_{k,l=1}^{2} C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell *} C_L^{\chi_l^+\widetilde{\ell}_a\nu_\ell} \times \\ \times \left[C_R^{\chi_k^+\chi H^- *}C_R^{\chi_l^+\chi H^-}\left\{-\left(s-m_\chi^2\right)\mathcal{T}_1^u + m^2_{\widetilde{\ell}_a}\left(m^2_{H^\pm}-m^2_\chi\right)\mathcal{T}_0^u\right\} - \right. \\ - m_\chi\left(m_{\chi_k^\pm}C_L^{\chi_k^+\chi H^- *}C_R^{\chi_l^+\chi H^-} + m_{\chi_l^\pm}C_R^{\chi_k^+\chi H^- *}C_L^{\chi_l^+\chi H^-}\right) \times \\ \times \left\{-\mathcal{T}_1^u + m^2_{\widetilde{\ell}_a}\mathcal{T}_0^u\right\} + \\ \left. + C_L^{\chi_k^+\chi H^- *}C_L^{\chi_l^+\chi H^-} m_{\chi_k^\pm} m_{\chi_l^\pm}\left\{\mathcal{T}_1^u + \left(s-m^2_{H^\pm}-m^2_{\widetilde{\ell}_a}\right)\mathcal{T}_0^u\right\}\right]; \quad (4.131)$$

- lepton ($\ell$) - sneutrino ($\widetilde{\nu}_\ell$) interference:

$$\widetilde{w}^{(\ell-\widetilde{\nu})}_{H^-\nu_\ell} = 4\,\mathrm{Re}\,\frac{C_S^{\widetilde{\nu}_\ell\widetilde{\ell}_a H^+}C_L^{\chi\widetilde{\nu}_\ell\nu_\ell *}C_S^{\nu_\ell\ell H^+ *}}{s-m_\ell^2} \times \\ \times\left[C_L^{\chi\widetilde{\ell}_a\ell}m_\ell\left\{-1-\left(m^2_{\widetilde{\nu}_\ell}-m^2_\chi\right)\mathcal{F}^t\right\} + C_R^{\chi\widetilde{\ell}_a\ell}m_\chi\left(s-m^2_{H^\pm}\right)\mathcal{F}^t\right]; \quad (4.132)$$

- lepton ($\ell$) - chargino ($\chi_k^\pm$) interference:

$$\widetilde{w}^{(\ell-\chi^\pm)}_{H^-\nu_\ell} = 4\,\mathrm{Re}\sum_{k=1}^{2}\frac{C_S^{\nu_\ell\ell H^+ *}C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell}}{s-m_\ell^2}\left[C_R^{\chi\widetilde{\ell}_a\ell}C_R^{\chi_k^+\chi H^- *}\left\{s+\left(sm^2_{\chi_k^\pm}-m^2_{H^\pm}m^2_{\widetilde{\ell}_a}\right)\mathcal{F}^u\right\} + \right. \\ + C_L^{\chi\widetilde{\ell}_a\ell}C_L^{\chi_k^+\chi H^- *}m_\ell m_{\chi_k^\pm} \times \quad (4.133) \\ \times\left\{1+\left(s+m^2_{\chi_k^\pm}-m^2_{\widetilde{\ell}_a}-m^2_{H^\pm}\right)\mathcal{F}^u\right\} + \\ + C_R^{\chi\widetilde{\ell}_a\ell}C_L^{\chi_k^+\chi H^- *}m_\chi m_{\chi_k^\pm}\left(s-m^2_{H^\pm}\right)\mathcal{F}^u + \\ \left. + C_L^{\chi\widetilde{\ell}_a\ell}C_R^{\chi_k^+\chi H^- *}m_\ell m_\chi\left\{1+\left(m^2_{\chi_k^\pm}-m^2_{\widetilde{\ell}_a}\right)\mathcal{F}^u\right\}\right];$$

• <u>sneutrino ($\widetilde{\nu}$) - chargino ($\chi_k^\pm$) interference:</u>

$$
\begin{aligned}
\widetilde{w}_{H^-\nu_\ell}^{(\widetilde{\nu}-\chi^\pm)} = -2\,\mathrm{Re}\sum_{k=1}^{2} & C_S^{\widetilde{\nu}_\ell^*\widetilde{\ell}_a H^{+*}} C_L^{\chi\widetilde{\nu}_\ell\nu_\ell} C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell} \times \\
& \times \Bigg[ -\left( m_{\chi_k^\pm} C_L^{\chi_k^+\chi H^{-*}} + m_\chi C_R^{\chi_k^+\chi H^{-*}} \right) \mathcal{Y}_1 + \\
& \quad + \Bigg[ m_{\chi_k^\pm} C_L^{\chi_k^+\chi H^{-*}} \left\{ 1 - \frac{1}{s}\left( m_{\widetilde{\ell}_a}^2 - m_\chi^2 \right) \right\} - \qquad (4.134) \\
& \quad - m_\chi C_R^{\chi_k^+\chi H^{-*}} \left\{ 1 + \frac{1}{s}\left( m_{\widetilde{\ell}_a}^2 - m_\chi^2 \right) \right\} \Bigg] \left( s - m_{H^\pm}^2 \right) \mathcal{Y}_0 \Bigg].
\end{aligned}
$$

## 5. Numerical analysis

In this section we present some numerical examples to illustrate the effect of the neutralino-slepton coannihilation. We will work in the framework of the CMSSM where the effect considered in this paper is particularly important. In computing the neutralino WIMP relic abundance we will employ the exact expressions for the cross-sections of neutralino pair-annihilation derived in [4] and the neutralino-slepton coannihilation ones listed above. To generate mass spectra we will use the package SUSPECT (v.2.05) [31], which includes full one-loop radiative corrections to sfermion masses as well as to the effective potential.

We begin by presenting in figure 1 contours of the relic abundance $\Omega_\chi h^2$ in the plane $(m_{1/2}, m_0)$ for $\tan\beta = 10$ (left window) and 40 (right window), and for $A_0 = 0$ and $\mu > 0$. The solid (dashed) curves correspond to including (neglecting) the coannihilation effect. The light orange region of $\Omega_\chi h^2 > 0.3$ is inconsistent with the age of the Universe while the green band corresponding to $0.1 < \Omega_\chi h^2 < 0.2$ is favored by direct measurements of the dark matter component in the Universe. For the sake of clarity, we only denote (in red) the regions of the plane where the lighter stau $\widetilde{\tau}_1$ is the LSP (and in some part of it one of the Higgs boson mass-square is negative), as well as those excluded by the LEP limit on the lightest chargino but no other experimental bounds. In particular, we do not indicate the regions inconsistent with the lightest Higgs boson mass bound from LEP, nor with $\mathrm{BR}(B \to X_s\gamma)$. (Their effect has been presented in [19] with updates in [32].) We only note that these bounds, if taken at face value, exclude the favored regions of $0.1 < \Omega_\chi h^2 < 0.2$ (green bands) of $m_{1/2} \lesssim 320$ GeV ($\tan\beta = 10$) and $m_{1/2} \lesssim 550$ GeV ($\tan\beta = 40$). In reality, theoretical uncertainties may considerably weaken these bounds, but in any case, it is clear that the remaining region is allowed mostly only by the coannihilation effect.

Note that the narrow band opened up by coannihilation eventually ends at the boundary of equal neutralino and stau masses. This is in qualitative agreement with the results obtained in [14, 23, 24] using the usual partial wave expansion but not with the more recent analysis [16] where the cosmologically allowed region appears to lie basically parallel to the boundary even at very large $m_{1/2}$. We will come back to discussing the upper limit on $m_\chi$ below.

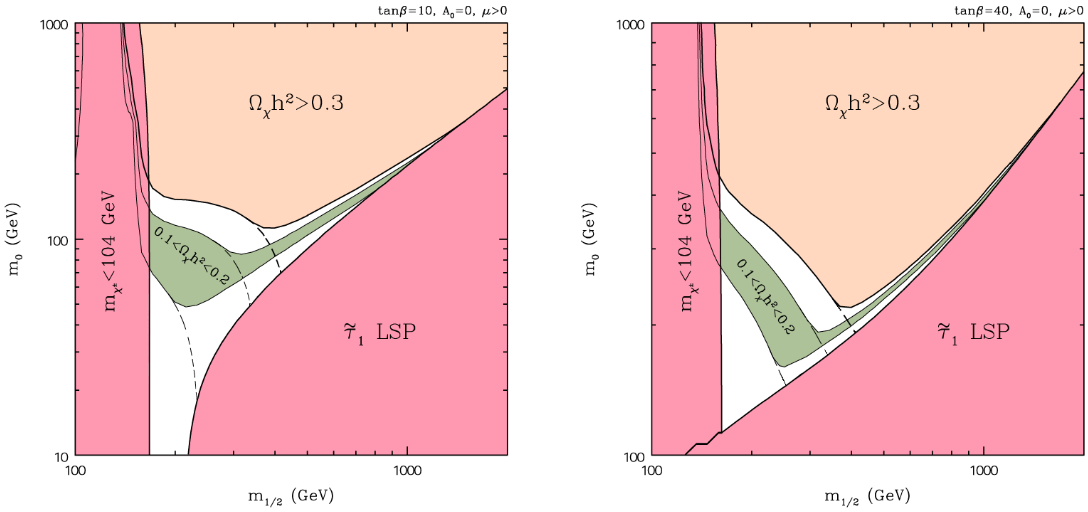


**Figure 1:** Contours of $\Omega_\chi h^2$ in the plane $(m_{1/2}, m_0)$ for $\tan\beta = 10$ (left window) and $\tan\beta = 40$ (right window), and for $A_0 = 0$, $\mu > 0$, $m_t^{\rm pole} = 175$ GeV and $m_b(m_b)_{\rm SM}^{\overline{\rm MS}} = 4.20$ GeV. The red regions bands are excluded by chargino searches at LEP and corresponds to the lighter stau being the LSP. The light orange regions of $\Omega_\chi h^2 > 0.3$ are excluded by cosmology while the narrow green bands correspond to the expected range $0.1 < \Omega_\chi h^2 < 0.2$. Also shown are the semi-oval contours of $\Omega_\chi h^2$ in the absence of coannihilation.

The effect of coannihilation becomes dramatic when the two coannihilating particles are nearly degenerate in mass, but can be significant even for the mass difference of some $20 - 40$ GeV. This is illustrated in figure 2 where the relic abundance $\Omega_\chi h^2$ is plotted as a function of $\Delta m = m_\chi - m_{\tilde{\tau}_1}$ for $\tan\beta = 10$ (left window) and 40 (right window) at a fixed value of $m_{1/2} = 500$ GeV. Also marked are the cosmologically excluded ($\Omega_\chi h^2 > 0.3$) and favored ($0.1 < \Omega_\chi h^2 < 0.2$) regions.

In figure 3 we present the total as well as the individual contributions to the quantity $J(x_f) \equiv \int_0^{x_f} dx \langle \sigma v_{\rm Møl}\rangle(x)$ from the various classes of processes listed in table 1. (Roughly, $\Omega_\chi h^2 \sim 1/J(x_f)$ - see, e.g. [4].) This is done for a slice of constant $m_{1/2} = 500$ GeV as a function of $\Delta m/m_\chi = (m_\chi - m_{\tilde{\tau}_1})/m_\chi$ for the two choices of $\tan\beta = 10, 40$ of figure 1. At a smaller mass difference it is the slepton-slepton annihilation into lepton pairs (mostly $\tilde{\tau}_1\tilde{\tau}_1 \to \tau\tau$) that appears to be dominant but it drops quickly and at larger $\Delta m/m_\chi$ it is overtaken by the neutralino-slepton coannihilation channel (mostly the $\gamma\tau$ final state). Here our conclusions qualitatively agree with [23].

As mentioned in the Introduction, [23] contains the only set of analytic expresssions for the neutralino-slepton coannihilation that is at present available in the literature. As noted by the authors, the expressions given there are approximate as they were derived using partial wave expansion. They also become less reliable at large values of $\tan\beta \gtrsim 20$ [24] because the effects of $h_\tau$ and of the $\tilde{\tau}_1 - \tilde{\tau}_2$ mixing, and in some channels of the mass of the $\tau$, were neglected. We have made an attempt at improving the expressions of [23] by

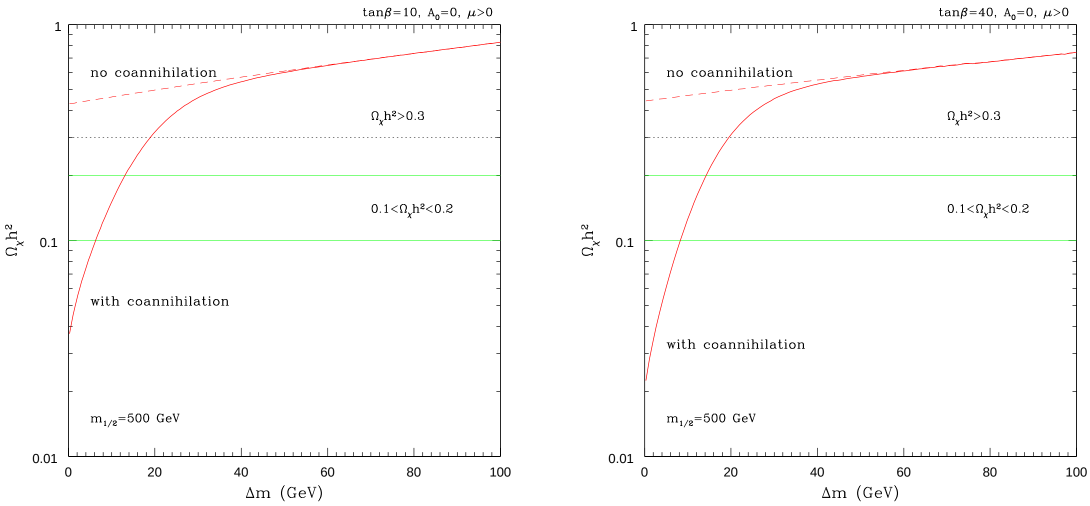


**Figure 2:** The relic abundance $\Omega_\chi h^2$ with (solid) and without (dash) coannihilation *vs.* $\Delta m = m_\chi - m_{\tilde{\tau}_1}$ for $\tan\beta = 10$ (left window) and 40 (right window) at a fixed value of $m_{1/2} = 500$ GeV. Also marked are the cosmologically excluded ($\Omega_\chi h^2 > 0.3$) and favored ($0.1 < \Omega_\chi h^2 < 0.2$) regions.

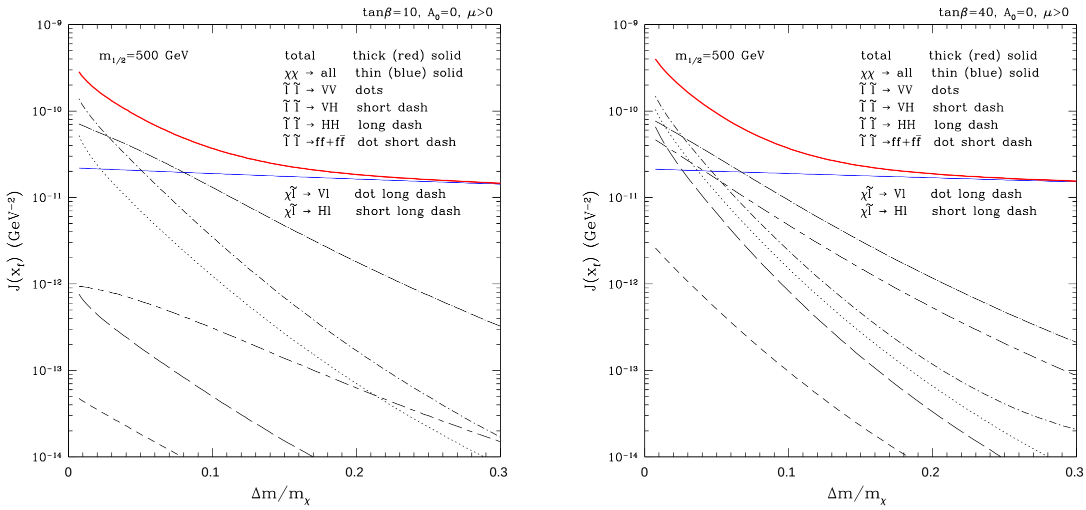


**Figure 3:** Total and partial contributions to $J(x_f)$ from the various classes of processes listed in table 1 as a function of $\Delta m/m_\chi = (m_\chi - m_{\tilde{\tau}_1})/m_\chi$ for the two choices of $\tan\beta = 10, 40$ of figure 1.

including in the propagators the widths of the gauge and Higgs boson s and the neutralinos; otherwise they become singular.

While keeping these points in mind, it is nevertheless interesting to compare them with the exact expressions derived here. We do this numerically in figure 4 for the cases

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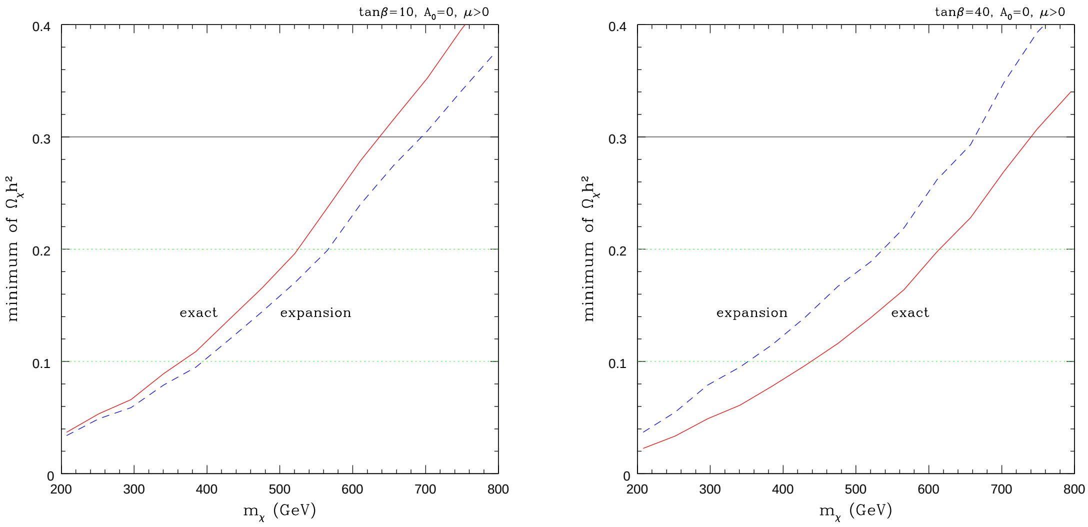


**Figure 4:** The minimum of $\Omega_\chi h^2$ (i.e. $\Omega_\chi h^2$ along the line $m_\chi = m_{\tilde{\tau}_1}$) as a function of $m_\chi$ for the two representative choices of figure 1. The solid (red) curve correspods to the exact neutralino-slepton coannihilation cross sections, while the dashed (blue) ones to the approximate ones of [23].

displayed in figure 1. Along the boundary of $m_\chi = m_{\tilde{\tau}_1}$ (and in fact in most of the $(m_{1/2}, m_0)$ plane) $m_\chi \simeq 0.44 m_{1/2} - 2.8 \sin 2\beta$ [33]. This allows us to plot the minimum value of $\Omega_\chi h^2$ along the boundary as a function of $m_\chi$. It is clear that $\Omega_\chi h^2$ increases with $m_\chi$ and at some point becomes inconsistent with $\Omega_\chi h^2 < 0.3$. The solid (red) curve correspond to the exact results while the dash (blue) ones were obtained by using the approximate expressions of [23]. For $\tan\beta = 10$ the upper limit from $\Omega_\chi h^2 < 0.3$ on $m_\chi$ is strengthened from $\sim 700$ GeV down to some 640 GeV. In contrast, at large $\tan\beta = 40$ (where, we repeat, the approximate expressions are not really applicable [23]), an upper bound becomes considerably weaker ($m_\chi \lesssim 765$ GeV) than what one would get by naively applying the expressions of [23]. The ranges of $m_\chi$ corresponding to the favored range $0.1 < \Omega_\chi h^2 < 0.2$ also shift accordingly.

## 6. Summary

The accuracy of determining the abundance of the dark matter in the Universe is continuously improving. This requires theoretical computations of the neutralino relic abundance to be performed with at least the same, if not better, level of precision, if one wants to reliably compare theoretical predictions with observations.

In this paper we have derived a full set of exact, analytic expressions for the neutralino-slepton coannihilation cross sections into all tree-level two-body final states. While these formulae are applicable in the framework of the general MSSM, they are of particular importance in the context of the Constrained MSSM. In this framework, which is often considered a "reference" SUSY model and is thus of much interest to the community, much of the allowed regions are a result of the neutralino-slepton coannihilation. Our results should help in allowing one to determine these regions more precisely.

## Acknowledgments

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## A. Lagrangian terms and couplings

In this appendix, we define the couplings which appear in the main text and which have not been defined in [4]. We follow the same conventions and notation as in that paper.

**Slepton - slepton - Higgs.**

$$\mathcal{L} = \left(C^{\widetilde{\ell}_b^* \widetilde{\ell}_a h} h + C^{\widetilde{\ell}_b^* \widetilde{\ell}_a H} H + C^{\widetilde{\ell}_b^* \widetilde{\ell}_a A} A\right) \widetilde{\ell}_b^* \widetilde{\ell}_a + \left[C^{\widetilde{\nu}_\ell^* \widetilde{\ell}_a H^+} \widetilde{\nu}_\ell^* \widetilde{\ell}_a H^+ + \text{h.c.}\right], \tag{A.1}$$

where

$$\begin{aligned}
C^{\widetilde{\ell}_b^* \widetilde{\ell}_a r} &= C^r_{LL} \left(\widetilde{V}_\ell^\dagger\right)^*_{1b} \left(\widetilde{V}_\ell^\dagger\right)_{1a} + C^r_{RR} \left(\widetilde{V}_\ell^\dagger\right)^*_{2b} \left(\widetilde{V}_\ell^\dagger\right)_{2a} + \\
&\quad + C^r_{LR} \left(\widetilde{V}_\ell^\dagger\right)^*_{1b} \left(\widetilde{V}_\ell^\dagger\right)_{2a} + C^r_{RL} \left(\widetilde{V}_\ell^\dagger\right)^*_{2b} \left(\widetilde{V}_\ell^\dagger\right)_{1a}, \qquad (r = h, H, A), \\
C^{\widetilde{\nu}_\ell^* \widetilde{\ell}_a H^+} &= C^{H^+}_{LL} \left(\widetilde{V}_\ell^\dagger\right)_{1a} + C^{H^+}_{LR} \left(\widetilde{V}_\ell^\dagger\right)_{2a},
\end{aligned} \tag{A.2}$$

with

$$\begin{aligned}
C^h_{LL} &= -\frac{g m_Z}{\cos\theta_W} \left(\frac{1}{2} - \sin^2\theta_W\right) \sin(\alpha+\beta) + \frac{g m_\ell^2}{m_W \cos\beta} \sin\alpha, \\
C^h_{RR} &= -\frac{g m_Z}{\cos\theta_W} \sin^2\theta_W \sin(\alpha+\beta) + \frac{g m_\ell^2}{m_W \cos\beta} \sin\alpha, \\
C^h_{LR} &= \frac{g m_\ell}{2 m_W \cos\beta} \left(A_\ell \sin\alpha + \mu \cos\alpha\right), \\
C^h_{RL} &= C^h_{LR}, \\
C^H_{LL} &= \frac{g m_Z}{\cos\theta_W} \left(\frac{1}{2} - \sin^2\theta_W\right) \cos(\alpha+\beta) - \frac{g m_\ell^2}{m_W \cos\beta} \cos\alpha, \\
C^H_{RR} &= \frac{g m_Z}{\cos\theta_W} \sin^2\theta_W \cos(\alpha+\beta) - \frac{g m_\ell^2}{m_W \cos\beta} \cos\alpha, \\
C^H_{LR} &= -\frac{g m_\ell}{2 m_W \cos\beta} \Big(A_\ell \cos\alpha - \mu \sin\alpha\Big), \\
C^H_{RL} &= C^H_{LR}, \\
C^A_{LL} &= 0, \\
C^A_{RR} &= 0,
\end{aligned}$$

$$
\begin{aligned}
C^A_{LR} &= -\frac{igm_\ell}{2m_W}\Big(A_\ell \tan\beta + \mu\Big)\,,\\
C^A_{RL} &= -C^A_{LR}\,,\\
C^{H^+}_{LL} &= -\frac{gm_W}{\sqrt{2}}\left[\sin 2\beta - \frac{m_\ell^2}{m_W^2}\tan\beta\right],\\
C^{H^+}_{LR} &= \frac{gm_\ell}{\sqrt{2}m_W}\Big(A_\ell \tan\beta + \mu\Big)\,.
\end{aligned}
\tag{A.3}
$$

Note that, for $a = b$, the pseudoscalar coupling $C^{\widetilde{\ell}^*_b \widetilde{\ell}_a A}$ vanishes in the absence of CP violating phases: $C^{\widetilde{\ell}^*_a \widetilde{\ell}_a A} = 0$.

**Slepton - slepton - gauge.**

$$
\mathcal{L} = \left[iC^{\widetilde{\nu}^*_\ell \widetilde{\ell}_a W^+}\left(\widetilde{\nu}^*_\ell \;\overleftrightarrow{\partial}^\mu\; \widetilde{\ell}_a\right)W^+_\mu + \text{h.c.}\right] + i\left(C^{\widetilde{\ell}^*_b \widetilde{\ell}_a Z} Z_\mu\right)\left(\widetilde{\ell}^*_b \;\overleftrightarrow{\partial}^\mu\; \widetilde{\ell}_a\right), \tag{A.4}
$$

where

$$
C^{\widetilde{\nu}^*_\ell \widetilde{\ell}_a W^+} = -\frac{g}{\sqrt{2}}\left(\widetilde{V}^\dagger_\ell\right)_{1a}, \tag{A.5}
$$

$$
\begin{aligned}
C^{\widetilde{\ell}^*_b \widetilde{\ell}_a Z} = -\frac{g}{\cos\theta_W}\Bigg[&\left(-\frac{1}{2} + \sin^2\theta_W\right)\left(\widetilde{V}^\dagger_\ell\right)^*_{1b}\left(\widetilde{V}^\dagger_\ell\right)_{1a} + \\
&+ \sin^2\theta_W \left(\widetilde{V}^\dagger_\ell\right)^*_{2b}\left(\widetilde{V}^\dagger_\ell\right)_{2a}\Bigg].
\end{aligned}
\tag{A.6}
$$

**Slepton - slepton - Higgs - Higgs.**

$$
\begin{aligned}
\mathcal{L} = \Bigg[&\frac{1}{2}C^{\widetilde{\ell}^*_b \widetilde{\ell}_a hh} h^2 + \frac{1}{2}C^{\widetilde{\ell}^*_b \widetilde{\ell}_a HH} H^2 + C^{\widetilde{\ell}^*_b \widetilde{\ell}_a hH} hH + \\
&+ \frac{1}{2}C^{\widetilde{\ell}^*_b \widetilde{\ell}_a AA} A^2 + C^{\widetilde{\ell}^*_b \widetilde{\ell}_a H^+H^-} H^+H^-\Bigg]\widetilde{\ell}^*_b \widetilde{\ell}_a\,,
\end{aligned}
\tag{A.7}
$$

where

$$
\begin{aligned}
C^{\widetilde{\ell}^*_b \widetilde{\ell}_a X} &= C^X_{LL}\left(\widetilde{V}^\dagger_\ell\right)^*_{1b}\left(\widetilde{V}^\dagger_\ell\right)_{1a} + C^X_{RR}\left(\widetilde{V}^\dagger_\ell\right)^*_{2b}\left(\widetilde{V}^\dagger_\ell\right)_{2a}\,,\\
(X &= hh, hH, AA, H^+H^-)\,,
\end{aligned}
\tag{A.8}
$$

and

$$
\begin{aligned}
C^X_{LL} &= \frac{g^2}{2}\left[C^X_1\left(\frac{-1 + 2\sin^2\theta_W}{2\cos^2\theta_W}\right) - \frac{m_\ell^2}{m_W^2}C^X_2\right],\\
C^X_{RR} &= \frac{g^2}{2}\left[-\tan^2\theta_W C^X_1 - \frac{m_\ell^2}{m_W^2}C^X_2\right],\\
C^{H^+H^-}_{LL} &= \frac{g^2\cos 2\beta}{4\cos^2\theta_W}\,,\\
C^{H^+H^-}_{LR} &= -\frac{g^2}{2}\left[\cos 2\beta \tan^2\theta_W + \frac{m_\ell^2}{m_W^2}\tan^2\beta\right],
\end{aligned}
\tag{A.9}
$$

with $C_1^X$ and $C_2^X$ $(X = hh, HH, hH, AA)$ in the following

$$\begin{aligned} C_1^{hh} &= \cos 2\alpha\,,\\ C_2^{hh} &= \frac{\sin^2\alpha}{\cos^2\beta}\,,\\ C_1^{HH} &= -\cos 2\alpha\,,\\ C_2^{HH} &= \frac{\cos^2\alpha}{\cos^2\beta}\,,\\ C_1^{hH} &= \sin 2\alpha\,,\\ C_2^{hH} &= -\frac{\sin 2\alpha}{2\cos^2\beta}\,,\\ C_1^{AA} &= \cos 2\beta\,,\\ C_2^{AA} &= \tan^2\beta\,. \end{aligned} \tag{A.10}$$

**Slepton - slepton - gauge - gauge.**

$$\mathcal{L} = \left[\frac{1}{2}C^{\widetilde{\ell}_b^*\widetilde{\ell}_a ZZ} Z_\mu Z^\mu + C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z\gamma} Z_\mu A^\mu + C^{\widetilde{\ell}_b^*\widetilde{\ell}_a WW} W_\mu^+ W^{-\mu}\right]\widetilde{\ell}_b^*\widetilde{\ell}_a\,, \tag{A.11}$$

where

$$\begin{aligned} C^{\widetilde{\ell}_b^*\widetilde{\ell}_a ZZ} &= \frac{2g^2}{\cos^2\theta_W}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2 \left(\widetilde{V}_\ell^\dagger\right)^*_{1b}\left(\widetilde{V}_\ell^\dagger\right)_{1a} + \sin^4\theta_W\left(\widetilde{V}_\ell^\dagger\right)^*_{2b}(\widetilde{V}_\ell^\dagger)_{2a}\right],\\ C^{\widetilde{\ell}_b^*\widetilde{\ell}_a Z\gamma} &= -2g^2\tan\theta_W\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)\left(\widetilde{V}_\ell^\dagger\right)^*_{1b}\left(\widetilde{V}_\ell^\dagger\right)_{1a} + \sin^2\theta_W\left(\widetilde{V}_\ell^\dagger\right)^*_{2b}\left(\widetilde{V}_\ell^\dagger\right)_{2a}\right],\\ C^{\widetilde{\ell}_b^*\widetilde{\ell}_a WW} &= \frac{1}{2}g^2\left(\widetilde{V}_\ell^\dagger\right)^*_{1b}\left(\widetilde{V}_\ell^\dagger\right)_{1a}. \end{aligned} \tag{A.12}$$

Some more couplings including the neutrinos:

$$\begin{aligned} \mathcal{L} &= C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell}\widetilde{\ell}_a\bar{\nu}_\ell(1+\gamma_5)\chi_k^+ + C_L^{\ell\nu_\ell H^-}\bar{\ell}(1-\gamma_5)\nu_\ell H^+\\ &\quad + C_L^{\ell\nu_\ell W}\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\ell W_\mu^+ + \text{h.c.}\,, \end{aligned} \tag{A.13}$$

where

$$\begin{aligned} C_L^{\chi_k^+\widetilde{\ell}_a\nu_\ell} &= -\frac{1}{2}g\left[U_{k1}\left(\widetilde{V}_\ell^\dagger\right)_{1a} - \frac{m_\ell}{\sqrt{2}m_W\cos\beta}U_{k2}\left(\widetilde{V}_\ell^\dagger\right)_{2a}\right],\\ C_L^{\ell\nu_\ell H^-} &= \frac{g m_\ell\tan\beta}{2\sqrt{2}m_W}\,,\\ C_L^{\ell\nu_\ell W} &= -\frac{g}{2\sqrt{2}}\,. \end{aligned} \tag{A.14}$$

**Neutralino - lepton - slepton.**

$$\mathcal{L} = \sum_{\ell=e,\mu,\tau}\sum_{i=1}^{4}\sum_{a=1}^{2}\left\{\overline{\chi_i^0}\left(C_L^{\chi_i^0\widetilde{\ell}_a^*\ell}\frac{1-\gamma_5}{2} + C_R^{\chi_i^0\widetilde{\ell}_a^*\ell}\frac{1+\gamma_5}{2}\right)\ell\,\widetilde{\ell}_a^* + \text{h.c.}\right\}, \tag{A.15}$$

where

$$C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell} = -\frac{g}{\sqrt{2}} \left[ \left( \widetilde{V}_\ell \right)_{a1}^* \ell^{\chi_i^0 \ell L} + \left( \widetilde{V}_\ell \right)_{a2}^* \ell^{\chi_i^0 \ell R} \right], \tag{A.16}$$

$$C_R^{\chi_i^0 \widetilde{\ell}_a^* \ell} = -\frac{g}{\sqrt{2}} \left[ \left( \widetilde{V}_\ell \right)_{a1}^* r^{\chi_i^0 \ell L} + \left( \widetilde{V}_\ell \right)_{a2}^* r^{\chi_i^0 \ell R} \right]. \tag{A.17}$$

The couplings $\ell^{\chi_i^0 \ell L}$, $\ell^{\chi_i^0 \ell R}$, $r^{\chi_i^0 \ell L}$ and $r^{\chi_i^0 \ell R}$ are defined in [4]. Note that $\ell = e, \mu, \tau$ represents a charged lepton. We neglect generation mixing in the lepton sector. The charged slepton mass eigenstates $\widetilde{\ell}_a$ $(a = 1, 2)$ are related to the slepton gauge eigenstates $\widetilde{\ell}_L$ and $\widetilde{\ell}_R$ via

$$\widetilde{\ell}_a = \left( \widetilde{V}_\ell \right)_{a1} \widetilde{\ell}_L + \left( \widetilde{V}_\ell \right)_{a2} \widetilde{\ell}_R, \tag{A.18}$$

where $\widetilde{V}_\ell$ denotes a $2 \times 2$ matrix which diagonalizes the charged slepton mass matrix: $\widetilde{V}_\ell \, \mathcal{M}_{\widetilde{\ell}}^2 \, \widetilde{V}_\ell^\dagger = \mathrm{diag}(m_{\widetilde{\ell}_1}^2, m_{\widetilde{\ell}_2}^2)$. We also use $C_S^{\chi_i^0 \widetilde{\ell}_a^* \ell}$ and $C_P^{\chi_i^0 \widetilde{\ell}_a^* \ell}$ defined by

$$\begin{aligned} C_S^{\chi_i^0 \widetilde{\ell}_a^* \ell} &= \frac{1}{2} \left( C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell} + C_R^{\chi_i^0 \widetilde{\ell}_a^* \ell} \right), \\ C_P^{\chi_i^0 \widetilde{\ell}_a^* \ell} &= \frac{1}{2} \left( C_L^{\chi_i^0 \widetilde{\ell}_a^* \ell} - C_R^{\chi_i^0 \widetilde{\ell}_a^* \ell} \right). \end{aligned} \tag{A.19}$$

## B. Auxiliary functions

Here we give definitions for the auxiliary functions used in the text. The functions $\mathcal{F}^t$ and $\mathcal{F}^u$ are given by

$$\begin{aligned} \mathcal{F}^t(s, x_1, x_2, y_1, y_2, z) &= \frac{1}{2F} \ln \left| \frac{D_0 - D_1 + F - z}{D_0 - D_1 - F - z} \right|, \\ \mathcal{F}^u(s, x_1, x_2, y_1, y_2, z) &= \frac{1}{2F} \ln \left| \frac{D_0 + D_1 + F - z}{D_0 + D_1 - F - z} \right|, \end{aligned} \tag{B.1}$$

where

$$\begin{aligned} D_0 &= -\frac{s}{2} + \frac{x_1 + x_2 + y_1 + y_2}{2}, \\ D_1 &= \frac{(x_1 - x_2)(y_1 - y_2)}{2s}, \\ F &= \frac{1}{2} \sqrt{s - (\sqrt{x_1} + \sqrt{x_2})^2} \sqrt{1 - \frac{(\sqrt{x_1} - \sqrt{x_2})^2}{s}} \times \\ &\quad \times \sqrt{s - (\sqrt{y_1} + \sqrt{y_2})^2} \sqrt{1 - \frac{(\sqrt{y_1} - \sqrt{y_2})^2}{s}}. \end{aligned} \tag{B.2}$$

For $y_1 = y_2$, $D_1$ vanishes so that the two functions $\mathcal{F}^t$ and $\mathcal{F}^u$ reduce to the same function which is denoted by $\mathcal{F}$.

The functions $\mathcal{T}_i^t$ and $\mathcal{T}_i^u$ are obtained from $\mathcal{T}_i$ in [4] by the following replacements

$$\begin{aligned} \mathcal{T}_i^t(s, x_1, x_2, y_1, y_2, z_1, z_2) &= \mathcal{T}_i \left( D \longrightarrow D_0 - D_1, \mathcal{F} \longrightarrow \mathcal{F}^t \right), \\ \mathcal{T}_i^u(s, x_1, x_2, y_1, y_2, z_1, z_2) &= \mathcal{T}_i \left( D \longrightarrow D_0 + D_1, \mathcal{F} \longrightarrow \mathcal{F}^u \right). \end{aligned} \tag{B.3}$$



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The functions $\mathcal{T}_i$ in the right-hand side represent those defined in [4], where $F$ in [4] should be replaced with $F$ in this appendix. For $y_1 = y_2$, the two functions $\mathcal{T}_i^t$ and $\mathcal{T}_i^u$ reduce to the same function which is denoted by $\mathcal{T}_i$. The expressions for the functions $\mathcal{Y}_i$ $(i = 0, 1, 2, 3, 4)$ are given by

$$\mathcal{Y}_0 = \frac{1}{z_1 + z_2 - 2D_0} \left[\mathcal{F}^t\,(s, x_1, x_2, y_1, y_2, z_1) + \mathcal{F}^u\,(s, x_1, x_2, y_1, y_2, z_2)\right], \tag{B.4}$$

$$\begin{aligned}\mathcal{Y}_1 = \frac{2}{z_1 + z_2 - 2D_0}\Big[&(z_1 - D_0 + D_1)\mathcal{F}^t(s, x_1, x_2, y_1, y_2, z_1) - \\ &- (z_2 - D_0 - D_1)\,\mathcal{F}^u\,(s, x_1, x_2, y_1, y_2, z_2)\Big],\end{aligned} \tag{B.5}$$

$$\begin{aligned}\mathcal{Y}_2 = 1 + \frac{1}{z_1 + z_2 - 2D_0}\Big[&(z_1 + D_1)\,(z_1 - 2D_0 + D_1)\,\mathcal{F}^t\,(s, x_1, x_2, y_1, y_2, z_1) + \\ &+ (z_2 - D_1)\,(z_2 - 2D_0 - D_1)\,\mathcal{F}^u\,(s, x_1, x_2, y_1, y_2, z_2)\Big],\end{aligned} \tag{B.6}$$

$$\begin{aligned}\mathcal{Y}_3 = {} & 2\,(z_1 - z_2 + 2D_1) + \\ &+ \frac{2}{z_1 + z_2 - 2D_0}\Big[(z_1 + D_1)\,(z_1 - D_0 + D_1)(z_1 - 2D_0 + D_1)\,\mathcal{F}^t\,(s, x_1, x_2, y_1, y_2, z_1) - \\ &\qquad - (z_2 - D_1)\,(z_2 - D_0 - D_1)\,(z_2 - 2D_0 - D_1)\,\mathcal{F}^u \times \\ &\qquad \times\,(s, x_1, x_2, y_1, y_2, z_2)\Big],\end{aligned} \tag{B.7}$$

$$\begin{aligned}\mathcal{Y}_4 = {} & -D_0^2 + 3D_1^2 - D_0\,(z_1 + z_2) + 3D_1\,(z_1 - z_2) + \left(z_1^2 + z_2^2 - z_1\,z_2\right) + \frac{1}{3}F^2 + \\ &+ \frac{1}{z_1 + z_2 - 2D_0}\Big[(z_1 + D_1)^2\,(z_1 - 2D_0 + D_1)^2\,\mathcal{F}^t\,(s, x_1, x_2, y_1, y_2, z_1) + \\ &\qquad + (z_2 - D_1)^2\,(z_2 - 2D_0 - D_1)^2\,\mathcal{F}^u\,(s, x_1, x_2, y_1, y_2, z_2)\Big],\end{aligned} \tag{B.8}$$

where $\mathcal{Y}_i = \mathcal{Y}_i(s, x_1, x_2, y_1, y_2, z_1, z_2)$.

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